SAWING THROUGH THE LAYERS

A Lesson Involving Modeling with Linear Functions

INTRODUCE

Whole Class

Student Page
The Rope Problem: Getting Started

Student Pages
Cutting the Rope

Materials
- Colored pencils

- Discuss terminology (bends, layers, etc) and answers to questions 1-3. These terms and questions establish a foundation for the layers (rope) cutting investigation. Students examine the number of pieces created when a rope forms snake-like patterns with various layers, and is then cut. Be sure students understand that the rope must not overlap itself. (That would be a different investigation.)

  What would a snake-like pattern with one layer look like? Two layers? Three layers? Four layers? Invite students to demonstrate by making sketches.

- Select a colored pencil. Complete Table 1 so that students understand how to record the number of cuts and number of pieces for a rope that has one layer. Have students generate a rule for the number of pieces when there is one layer.

  Though every table is almost exactly the same, what is different about the next three? Each table is used to record the number of cuts and pieces for a different number of layers.

- Students graph the information from Table 1 using the same color as the table. Be sure they label the graph clearly.

- If needed, begin to complete Table 2 (using a second color), but refrain from doing too much of the investigation as a whole class because it will spoil the exploration.

EXPLORE

Partners

Student Pages
Cutting the Rope

- Students complete the tables and answer questions as they work toward finding a general formula that will give the number of pieces obtained for any number of layers and cuts.

SUMMARIZE

Whole Class

Student Pages
Cutting the Rope

- Students share patterns related to their tables, equations, and graphs.

  What is the same about the graphs? They are all lines. The context suggests a discrete graph, but drawing a trend line is good for the purpose of analysis. All these lines have a *y*-intercept at the point (0, 1). Since it is a point of intersection for all of the lines, this ordered pair also represents the solution if these equations are thought of as a system, and describes the fact that no matter the number of layers, when there are 0 cuts, there is one piece of rope.

  What is different about the graphs? Some are steeper/flatter than others. In other words, they have different slopes. The more layers in the rope, the steeper the slope.

  What does the y-intercept represent for each graph? This shows that when there are no cuts, there is one piece.

  What does the slope mean? The slope is the same as the number of layers.

  What is a rule for the number of pieces for any number of layers (q) and any number of cuts (c)? $p = qc + 1$.

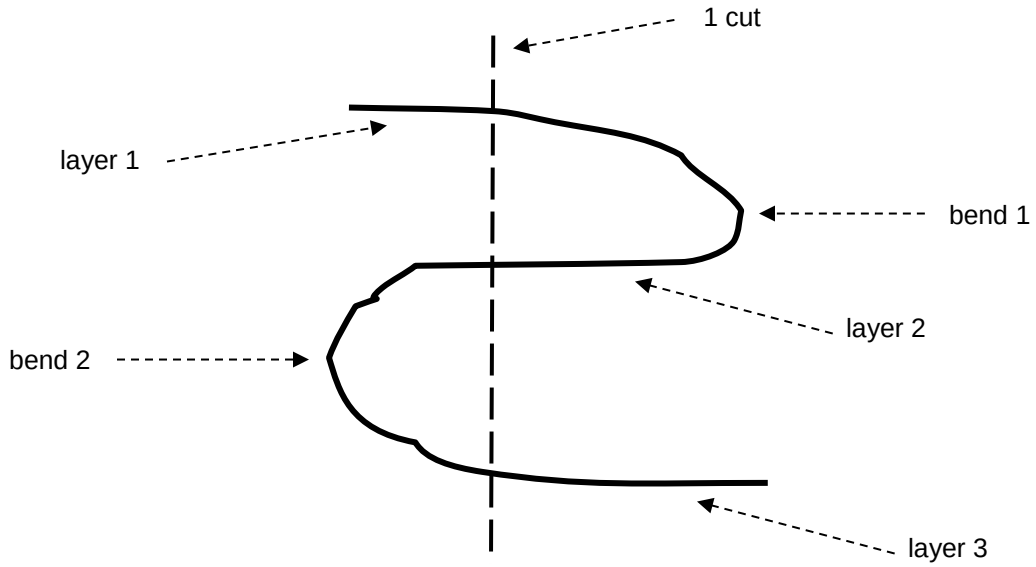
Adapted especially for NCTM 2016 - from *MathLinks*: Grade 8 (Lesson 9.1)

THE ROPE PROBLEM: GETTING STARTED

Many situations can be modeled using mathematics. In this problem, you will use the pictures, numbers, symbols and words to explore patterns that arise from cutting a rope.

- Start with **one** long piece of rope with three "layers" and two "bends."
- One vertical cut is made through all three layers as shown.

1. After one cut, how many pieces of rope are there? _____



2. Draw a 2nd vertical line that cuts through each layer again.

   How many pieces of rope are there now? _____

3. Draw a 3rd vertical line that cuts through each layer again.

   How many pieces of rope are there now? _____

CUTTING THE ROPE

Explore cutting the rope for different numbers of layers and cuts.

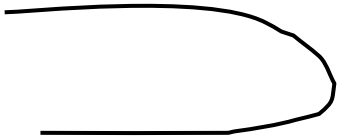
1. # of layers (q) = 1

# of cuts (c)	# of pieces (p)

Rule for any number of cuts:

p =

2. # of layers (q) = 2



# of cuts (c)	# of pieces (p)

Rule for any number of cuts:

p =

3. # of layers (q) = 3

# of cuts (c)	# of pieces (p)

Rule for any number of cuts:

p =

4. # of layers (q) = 4

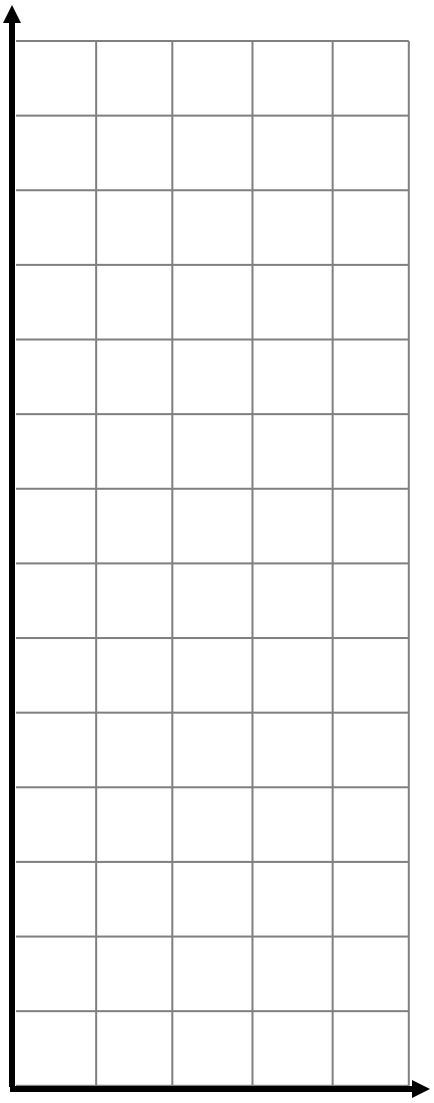
# of cuts (c)	# of pieces (p)

Rule for any number of cuts:

p =

CUTTING THE ROPE (Continued)

5. For each table, plot the points on the grid.
   Graph each set of points with a different color.
   Label and scale the axes appropriately.



6. What is the same about each graph?

7. What is different?

8. What does the *y*-intercept represent for each graph in terms of the cutting rope experiment?

9. What does the slope represent for each graph?

10. Looking at all four tables, write a rule that can be used to find the total number of pieces (p) for any number of layers (q) and any number of cuts (c):

    p = _____