Name______________________________ Period __________ Date _____________

7-5
STUDENT PACKET

MATHLINKS: GRADE 7
STUDENT PACKET 5
RATIONAL NUMBERS: MULTIPLICATION AND DIVISION 1

5.1 Multiplication: Models 1
- Explore the meaning of integer multiplication.
- Develop rules for integer multiplication using the counter and temperature change models.
- Extend rules for multiplication to non-integer rational numbers using arrows.

5.2 Multiplication and Division: Patterns 6
- Observe patterns that verify the rules for integer multiplication.
- Use the inverse relationship between multiplication and division to establish rules for integer division.
- Solve problems using integers.

5.3 Reasoning About Products and Quotients 12
- Write quotients of integers in different ways.
- Explore why division by zero is not defined.
- Prove that (-1)(-1) = 1.

5.4 Skill Builders, Vocabulary, and Review 21

WORD BANK

Word or Phrase	Definition or Description	Picture or Example
conjecture		
deductive reasoning		
inductive reasoning		
inverse operation		
product		
quotient		
rational number		

MULTIPLICATION: MODELS

Summary

We will develop integer multiplication rules using the counter and temperature change models. We will verify the rules for multiplication using arrows on a number line.

Goals

- Explore the meaning of integer multiplication.
- Develop rules for integer multiplication using the counter and temperature change models.
- Extend rules for multiplication to non-integer rational numbers using arrows.

Warmup

One way to think of multiplication is groups of objects with an equal number of objects in each group.

number of groups • number in each group = total number

1. Andre is purchasing 4 boxes of colored pencils. Each box has 6 pencils.

   a. Draw a diagram that shows Andre's purchase.

   b. How many groups of (boxes of) colored pencils did Andre purchase? ______

   c. How many pencils are in each group (box)? ___________

   d. Write a multiplication sentence to describe Andre's purchase. ___________

Build the following with positive and negative counters. Then draw a diagram to record your work.

2. 5 groups of 3 positive counters

3. 3 groups of 5 positive counters

4. 3 groups of 5 negative counters

MULTIPLICATION 1

Use the scripts in these templates to help you think through integer multiplication problems.
Do not write on this template.

Think about the counter model

(Begin with a work space that has a value equal to 0.)

- The first factor is positive. We will place _______ groups on the workspace.
- The second factor is __________ (pos / neg). Each group will contain _______ __________ (pos / neg) counter(s).
- The result is _______ __________ (pos / neg) counter(s).

Connect to the temperature change model

(Begin with temperature equal to zero degrees.)

- The first factor is positive. We will put _______ groups in the liquid.
- The second factor is __________ (pos / neg). Each group will contain _____ ___________________ (hot piece(s) / cold nugget(s)).
- The liquid is now ______ degree(s).

Compute each product. Think about the counter model and connect it to the temperature change model. Show your work by drawing positive (+) and negative (–) counters.

1. (2) • (4)	2. (2) • (-4)	3. (3) • (2)	4. (3) • (-2)

Refer to problems 1 - 4 above to answer problems 5 and 6.

5. Putting in groups of hot **p**ieces makes the liquid _______________.

   The product of a positive number and a positive number is a __________ number.

6. Putting in groups of cold **n**uggets makes the liquid _______________.

   The product of a positive number and a negative number is a __________ number.

MULTIPLICATION 2

Don't write on this template. Use it to help you think through integer multiplication problems.

Think about the counter model
(Begin with a work space that has a value equal to 0.)

- The first factor is _______. We will _______ _______ group(s) _______ the workspace.
  pos / neg place / remove in / from
- The second factor is _________. Each group will contain _______ _________ counter(s).
  pos / neg pos / neg
- I need at least _____ zero pairs to do this.
- The result is _________ _________ counter(s).
  pos / neg

Connect to the temperature change model
(Begin with temperature equal to zero degrees.)

- The first factor is _________. We will _________ _______ group(s) _________ the liquid.
  pos / neg put / remove in / from
- The second factor is _________. Each group will contain _____ ___________________.
  pos / neg hot **p**iece(s) / cold **n**ugget(s)
- Add zero pairs if needed.
- The liquid is now _____ degree(s).

Compute each product. Think about the counter model and connect it to the temperature change model. Show your work by drawing positive (+) and negative (–) counters.

1. (-2) • (4)	2. (-2) • (-4)	3. (-3) • (2)	4. (-3) • (-2)

Refer to problems 1 - 4 on page 2 and problems 1 - 4 above to answer problems 5 and 6.

5. Removing hot **p**ieces makes the liquid __________.

   The product of a negative number and a positive number is a ___________ number.

6. Removing cold **n**uggets makes the liquid ____________.

   The product of a negative number and a negative number is a ___________ number.

MULTIPLICATION PRACTICE

Compute. Refer to the counters/temperature scripts on pages 2 and 3 and draw diagrams as needed.

1. (4) • (-5)	2. (-4) • (3)	3. (-3) • (-5)
4. (-5) • (2)	5. (3) • (-1)	6. (-1) • (-2)
7. (8) • (5)	8. (-8) • (-5)	9. (7) • (-6)
10. (-3) • (-1)	11. (-9) • (-5)	12. (-6) • (-2)
13. (40) • (-5)	14. (-600) • (-3)	15. (25) • (50)

16. Summarize the rules for integer multiplication.

The product of two positive numbers is ____.

The product of two negative numbers is ______________________________________.

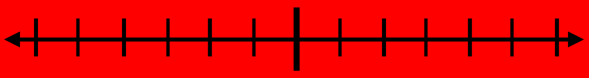
The product of a positive number and a negative number is ________________________.

The product of a negative number and a positive number is ________________________.

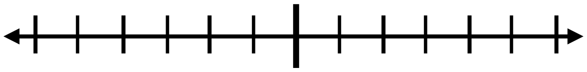
MULTIPLICATION USING ARROWS

There are several ways to explain multiplication using arrows. Do each multiplication problem below using arrows and explain your reasoning.

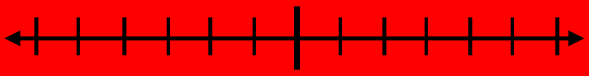
1. $(2)(3) =$ ______



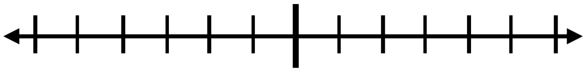
2. $(2)(-3) =$ ______



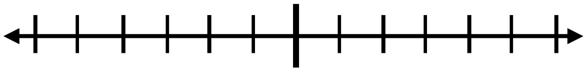
3. $(-2)(3) =$ ______



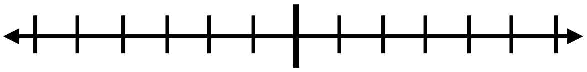
4. $(-2)(-3) =$ ______



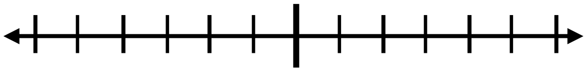
5. $(4)(0.25) =$ ______



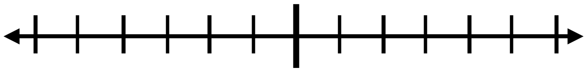
6. $(-2)\left(\frac{3}{4}\right) =$ ______



7. $\left(\frac{1}{2}\right)(-5) =$ ______



8. $\left(-\frac{1}{3}\right)(0.6) =$ ______



Don't write below this line.

MULTIPLICATION AND DIVISION: PATTERNS

Summary

We will use patterns and the inverse relationship between multiplication and division to establish sign rules for division. We will solve multiplication and division problems.

Goals

- Observe patterns that verify the rules for integer multiplication.
- Use the inverse relationship between multiplication and division to establish rules for integer division.
- Solve problems using integers.

Warmup

Complete the patterns started below by filling numbers in the blank spaces.

II I

						3	6	9		15
				-2	0	2		6	8	10
			-2	-1	0	1	2	3	4	
						0	0			
						-1				
			4							
							-8			

III IV

-5	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
5	**4**	**3**	**2**	**1**	**0**	**-1**	**-2**	**-3**	**-4**	**-5**

Use these strips when completing the following page.

USING A MULTIPLICATION TABLE

1. Describe at least two patterns formed by the numbers on the table from the warmup on page 6.

Cut out the two strips at the bottom of page 6. Then glue or tape them to the table on page 6 as directed by your teacher. The grid now serves as a multiplication table. Use it to find these products.

2. (4) • (3) = _____; located in region ____

3. (3) • (-1) = _____; located in region ____

4. (-3) • (5) = _____; located in region ____

5. (-4) • (-2) = _____; located in region ____

6. (-4) • (-3) = _____; located in region ____

7. (3) • (-5) = _____; located in region ____

Patterns for Multiplying Integers

8. The product of a positive number and a positive number is ________________.

9. The product of a positive number and a negative number is ________________.

10. The product of a negative number and a positive number is ________________.

11. The product of a negative number and a negative number is ________________.

Compute. Use the table on page 6 as needed.

12. (-5) • (4)

13. (3) • (-3)

14. (-4) • (-2)

Compute.

15. (-5) • (40)

16. (30) • (-30)

17. (-4) • (-200)

18. If 17 • 93 = 1,581, what is the product of -17 and 93? ____________

RELATING MULTIPLICATION AND DIVISION

1. Complete the patterns below. Use the table on page 6 as needed.

Multiplication Fact	**Corresponding Division Facts**
a. (5) • (4) = 20	(20) ÷ (4) = 5 (20) ÷ ( ) = 4
b. (4) • ( ) = -12	(-12) ÷ (4) = _____ (-12) ÷ ( ) = 4
c. ( ) • (2) = -8	(-8) ÷ (2) = _____ (-8) ÷ ( ) = 2
d. (-3) • ( ) = 6	(6) ÷ (-3) = _____ (6) ÷ ( ) = -3

Patterns for Dividing Integers

2. The quotient of a positive number and a positive number is _______________.	3. The quotient of a positive number and a negative number is _______________.
4. The quotient of a negative number and a positive number is _______________.	5. The quotient of a negative number and a negative number is _______________.

Compute. Use the table on page 6 as needed.

6. (-14) ÷ (7) = _____ because _____ • 7 = -14	7. (15) ÷ (-3) = _____ because _____ • _____ = 15	8. $\frac{-20}{-40}$ = _____ because _____ • (-4) = _____

9. How do the rules for multiplying integers compare to the rules for dividing integers?

MULTIPLICATION AND DIVISION PROBLEMS

1. A fish is swimming 15 feet below sea level.
   a. What number represents the fish's elevation when zero represents sea level? _____
   b. A dolphin is swimming 3 times as deep as the fish. What numerical expression represents the elevation that is 3 times the depth of the fish? __________
   c. What number represents the elevation of the dolphin? _____

2. During a cold week in Wisconsin, the temperature each day at noon in Fahrenheit was 4°, -6°, -1°, 3°, and 0°.
   a. What numerical expression can be used to find the average noontime temperature for the week?
   b. What was the average temperature at noon for the week? _____

3. During the same cold week in Wisconsin, the temperature each day at midnight in Fahrenheit was -4°, -6°, -10°, -3°, and -7°.
   a. What numerical expression can be used for the average midnight temperature for the week?
   b. What was the average temperature at midnight for the week? _____

4. The elevation of water in a lake rose 15 inches per month for 3 months and then dropped 2 feet per month for 4 months.
   a. What numerical expression can be used to describe the elevation change in inhes?
   b. After 7 months, was the elevation of the lake higher or lower than the starting elevation? _________________ By how much? _________________
   c. Write this gain or loss as a number. _____________

MULTIPLICATION AND DIVISION PRACTICE

Compute.

1. $7 \bullet (-3)$	2. $-9 \div 3$	3. $(-6) \bullet (-6)$
4. $(-5)(9)$	5. $(-40) \div 10$	6. $\frac{26}{13}$
7. $-\left(\frac{39}{13}\right)$	8. $-5(-12)$	9. $(-28) \div (-4)$
10. $\frac{77}{-11}$	11. $150 \bullet 20$	12. $(-4) \bullet (-40)$
13. $-7(-600)$	14. $(-320) \div (-80)$	15. $\frac{5600}{-80}$

16. Sylvia put some counters in her left hand and some more in her right hand and challenged her group to answer this question.

“The sum of the amounts in my hands is -15 and the product is 54. What do I have in each hand?” How should her group respond?

TARGET PRACTICE

Use a deck of cards with the picture cards removed. Red cards represent positive numbers. Black cards represent negative numbers.

Turn over 4 cards and record their values: ________ ________ ________ ________

For each problem below, write an expression using the four values above. Show your work. You may use addition, subtraction, multiplication, division, and any grouping symbols you know.

1. Write an expression with a value as close to 40 as possible.	2. Write an expression with a value as close to $-\frac{1}{2}$ as possible.
3. Write an expression with a value as large as possible.	4. Write an expression with a value as small as possible.

5. Compare your expression for one of the problems above with the expression of a classmate. Use equations or inequalities and words to describe which expression is closer to the target number.

REASONING ABOUT PRODUCTS AND QUOTIENTS

Summary

We will use inductive reasoning to make conjectures and deductive reasoning to draw conclusions based on accepted facts and logic. We will write quotients of integers in different forms. We will explore why division by zero is not defined. We will prove that (-1)(-1) = 1.

Goals

- Write quotients of integers in different ways.
- Explore why division by zero is not defined.
- Prove that (-1)(-1) = 1.

Warmup

1. Predict what comes next in the input-output table below.

x	0	1	2	3	4	5	6
y	0	2	4	6	8		

2. Describe in words why you think the outputs in the table above are correct.

3. Complete the input-output table below using the following rule: “To find each y-value, multiply each x-value by 2.1. Then round the product to the nearest whole number.

x	0	1	2	3	4	5	6
y		2					

Inductive reasoning is a form of reasoning in which the conclusion is supported by the evidence but is not proved.

A conjecture is a statement that is proposed to be true, but has not been proven to be true or to be false.

Deductive reasoning is a form of reasoning in which the conclusion is justified by an argument based on definitions, known facts, and accepted rules of logic.

4. In which problem(s) above did you:

a. use inductive reasoning? ___________

b. make a conjecture? ___________

c. use deductive reasoning? ___________

DETERMINING THE SIGN OF A PRODUCT OR QUOTIENT

Compute.

1. (-1) • (-1) • (1)	2. (-1) • (-1) • (-1)	3. 6(-5)(-2)
4. (-4)(-8)(-10)	5. (-2) • (-4) • (-5) • (10)	6. -2 • (-3) • (-4) • (-10)

7. Make conjectures about multiplying nonzero numbers.

   a. If there are an odd number of negative factors, the product is ____________________.
   positive / negative

   b. If there are an even number of negative factors, the product is ____________________.
   positive / negative

Compute, if possible.

8. -20 • (-30) • (-4) • (-200)	9. -12 • (-13) • (0) • (-210)	10. (-1)(-2)(-3)(-4)(-5)
11. -80 ÷ 10	12. -60 ÷ (-30)	13. 64 ÷ (-8)
14. $\frac{-45}{9}$	15. $\frac{-66}{-11}$	16. $-\left(\frac{36}{6}\right)$
17. -80 ÷ (-40)	18. $\frac{0}{3}$	19. $\frac{3}{0}$

20. Why is $\frac{-10}{5}$ not equal to $\frac{-10}{-5}$?

21. If the product of six integers is negative, at most how many of the integers can be negative?

MORE ABOUT THE SIGNS OF QUOTIENTS

Divide. If the quotient is not an integer, write it as a fraction in simplest form. Determine whether the quotient is positive or negative based upon integer division rules.

1. $\frac{10}{5}$	2. $\frac{-10}{-5}$	3. $\frac{-10}{5}$	4. $\frac{10}{-5}$	5. $-\left(\frac{10}{5}\right)$
6. $\frac{4}{16}$	7. $\frac{-4}{-16}$	8. $\frac{-4}{16}$	9. $\frac{4}{-16}$	10. $-\left(\frac{4}{16}\right)$

For the expressions below, a and b are positive integers. Circle all expressions that represent negative numbers.

$\frac{a}{b}$ $\frac{-a}{-b}$ $\frac{-a}{b}$ $\frac{a}{-b}$ $-\left(\frac{a}{b}\right)$

11. Bibi says that $\frac{-2}{-7}$ and $-\frac{2}{7}$ represent the same number. Is she correct? _____ Explain.

12. Julie says that $\frac{-12}{-42}$ and $\frac{12}{42}$ represent the same number. Is she correct? _____ Explain.

13. How do you know whether the quotient of two integers will be a positive number?

14. How do you know whether the quotient of two integers will be a negative number?

15. How do you know whether the quotient of two integers will be an integer?

RATIONAL NUMBERS

Rational numbers are quotients of integers, and the divisor cannot be zero.

Rational numbers can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

For example, neither -6 nor 4.5 are quotients of integers, but they can be expressed as quotients of integers:

$$-6 = \frac{-6}{1} \qquad \text{and} \qquad 4.5 = 4\frac{1}{2} = \frac{9}{2}.$$

1. Circle each rational number that is expressed as a quotient of integers in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Box each rational number that is NOT expressed as a quotient of integers in the above form.

$$\frac{3}{5} \quad \frac{9}{7} \quad \frac{6}{-11} \quad -4 \quad 0.5 \quad \frac{0}{5} \quad \frac{-2}{17} \quad \frac{-4}{-3} \quad 2\frac{1}{2} \quad 0 \quad \frac{3}{0}$$

2. Which of the above are not rational numbers? ______________________ Why?

Write each rational number below in at least three different equivalent forms.

3. $\frac{-8}{16}$	4. $\frac{-8}{-6}$	5. $-\frac{13}{5}$
6. $\frac{0}{13}$	7. $\frac{18}{-2}$	8. $\frac{-60}{20}$

RATIONAL NUMBERS (Continued)

Write each number below in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$ to show that they are rational.

9. -12	10. 4.75	11. $-3\frac{1}{2}$

12. Mark says that $-3\frac{1}{2}$ and $\frac{7}{-2}$ represent the same number. Is he correct? ______ Explain.

13. For the expressions below, a is a positive integer and b is a negative integer. Circle the expressions that represent negative numbers.

$\frac{a}{b}$ $\frac{-a}{-b}$ $\frac{-a}{b}$ $\frac{a}{-b}$ $-\left(\frac{a}{b}\right)$

Prove whether each of the following expressions represents a rational number or not.

14. $-17 \div (-3)$	15. $\frac{-12(-3)}{1-7}$
16. $\frac{5^2 - 25}{-2\frac{1}{4} - 3\frac{3}{4}}$	17. $\frac{5-18}{-\frac{1}{2} + \frac{2}{3} - \frac{1}{6}}$

WHAT HAPPENS IF?

Use examples and explain your reasoning for each problem.

Consider the quotient $\frac{c}{d}$, where c and d are both positive integers, and neither is equal to zero. Will the quotient get greater, lesser, or stay the same if…

1. Both c and d are multiplied by the same nonzero integer?

2. The value of c increases and the value of d decreases?

3. The same positive integer is added to both c and d?

4. The same nonzero integer is added to both c and d?

DIVISION BY ZERO?

Fill in each box below with the appropriate solution, if there is a unique solution.
Put an "X" in each box if the problem has no solution.
Circle each problem that has infinitely many solutions.

1.	$4 \bullet \square = 12$	$\longrightarrow$	$\begin{array}{r} \square \\ 4\overline{)\,12} \end{array}$
2.	$0 \bullet \square = 12$	$\longrightarrow$	$\begin{array}{r} \square \\ 0\overline{)\,12} \end{array}$
3.	$5 \bullet \square = 20$	$\longrightarrow$	$20 \div 5 = \square$
4.	$0 \bullet \square = 20$	$\longrightarrow$	$20 \div 0 = \square$
5.	$2 \bullet \square = 8$	$\longrightarrow$	$\frac{8}{2} = \square$
6.	$0 \bullet \square = 8$	$\longrightarrow$	$\frac{8}{0} = \square$
7.	$10 \bullet \square = 0$	$\longrightarrow$	$\frac{0}{10} = \square$
8.	$0 \bullet \square = 0$	$\longrightarrow$	$\frac{0}{0} = \square$

9. Why is it not possible to divide by zero?

Mathematically,
we say that
division by zero is
undefined.

PROVING (-1)(-1) = 1

1. By looking at many examples, we used inductive reasoning to make conjectures about the rules for signed number operations. Some conjectures about multiplication are:

   a. positive • positive → ______________ b. positive • negative → ______________

   c. negative • positive → _____________ d. negative • negative → _____________

2. We explored patterns and models to arrive at these conjectures. We performed multiplication in many examples. Does this prove that our conjectures are true? _____. Explain.

Find each product.

3. (1)(1) = _____	4. (-1)(1) = _____	5. (1)(-1) = _____

6. Which property justifies that the equations in problems 3 - 5 are true?

____

We will now prove using deductive reasoning that (-1)(-1) = 1. Justify each equation on the left with a property from the list on the right. You may use properties more than once.

	Equation	Property
7.	(-1)(-1 + 1) = (-1)(-1) + (-1)(1)	
8.	(-1)(0) = (-1)(-1) + (-1)(1)	
9.	0 = (-1)(-1) + (-1)(1)	
10.	0 = (-1)(-1) + (-1)	
11.	0 + 1 = (-1)(-1) + (-1) + 1	
12.	0 + 1 = (-1)(-1) + 0	
13.	1 = (-1)(-1)	

Properties

a. addition property of equality

b. additive identity property

c. additive inverse property

d. distributive property

e. multiplicative identity property

f. multiplication property of zero

POSTER PROBLEMS 1

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, etc. I am group member ____.
- Each group will start at a poster, #1, #2, #3, etc.. Our group start poster is # ____.
- Each group will have a different color marker. Our group marker is __________________.

Part 2: Answer the problems on the posters by following the directions of your teacher.

Part 3: Return to your seats.

1. Was the circled number on every poster the same? _____

   If not, recopy the directions below, use the start number given to you by your teacher, and rework the problem.

Step	**Start Number**

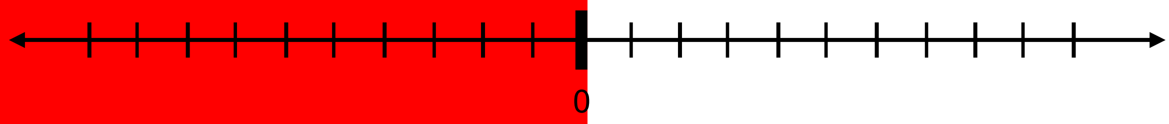
2. Why do you think the result is always the same?

SKILL BUILDERS, VOCABULARY, AND REVIEW

SKILL BUILDER 1

1. a. Graph the following numbers and their opposites on the number line below.

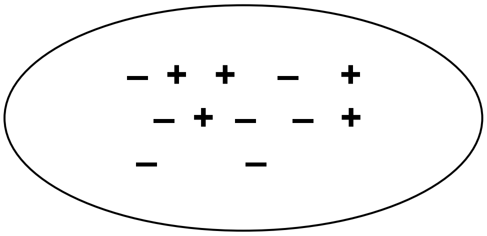
9 -4 0 -10



b. Use the symbols <, =, or > to write three number sentences comparing some of the numbers from the number line.

________________ ________________ ________________

2. What is the value of the collection of counters to the right? ______.



3. Explain the meaning of "zero pairs" in your own words.

Draw sketches for each value using positive (+) and negative (–) counters.

4. -4 using 8 counters	5. 5 using 11 counters	6. Zero using 10 counters

Compute.

7. $9 - 13$	8. $(-6) + 8$	9. $(-6.8) + (4)$
10. $(1.25) - (1.75)$	11. $\left(-\frac{5}{8}\right) + 5\frac{2}{3}$	12. $1\frac{2}{5} - 2\frac{1}{2}$

SKILL BUILDER 2

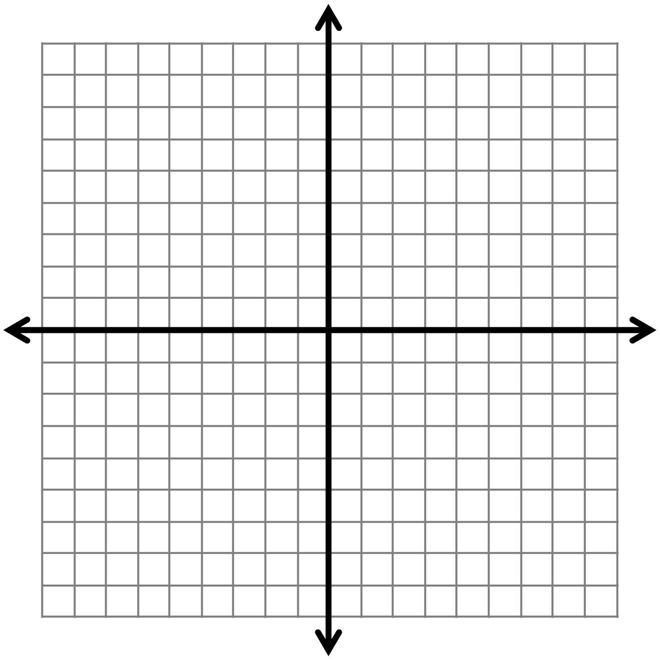
1. On a cross-country trip, Jake's family traveled 320 miles on Monday, 176 miles on Tuesday, and 578 miles on Wednesday. If the total distance they need to go is 1,241 miles, how much farther does Jake's family have to go?

2. An artist used 7.6 pounds of clay per month for 7 months. If the artist started with 140 pounds of clay, how much clay does the artist have after 7 months?

3. Draw and label the figure with vertices at:

   $P\ (3\frac{3}{4}, -\frac{1}{2})$ $L\ (2, -\frac{1}{2})$

   $D\ (3\frac{3}{4}, -4)$ $U\ (2, -4)$



4. Is $\overline{PL}$ vertical or horizontal?

5. Is $\overline{LU}$ vertical or horizontal?

Find the GCF and LCM of each pair of numbers.

Numbers	GCF	LCM
6. 18, 24		
7. 13, 5		
8. 12, 30		

SKILL BUILDER 3

1. Write an expression for “7 less than twice a number *n*.”

Evaluate.

2. $5a - (c + 4)$ for $a = 2$ and $c = 3$	3. $\frac{x+6}{y-2}$ for $x = 4$ and $y = 8$

4. Rewrite $24x - 6y$ as the product of a whole number and a difference of terms in two different ways.

5. Emiliano has 4 green marbles, 2 red marbles, and 6 white marbles. What is the ratio of the number of green marbles he has to the total number of marbles?

Write each ratio as a rate per one unit.

6. 80 miles per 4 hours (hint: how many miles per 1 hour?)	7. 8 meters per 5 seconds	8. 42 feet per 3 minutes

Write each ratio as a rate per one unit and determine which option is the better buy.

	Option 1	**Rate 1**	**Option 2**	**Rate 2**	**Better Buy**
9.	$32 for 8 pens		$27 for 9 pens		
10.	$88 for 11 books		$121.50 for 15 books		

11. If it took 5 hours to bake 25 apple pies, at that rate, how many apple pies could be baked in 23 hours?

SKILL BUILDER 4

1. Yasmine says that -3.6 is equivalent to $-3\frac{6}{10}$ and Jonathan says -3.6 is equivalent to $\frac{-18}{5}$. Who is correct? Explain.

Compute.

2. $(25-3)\ +7^2$	3. $(8-6)^2+1$

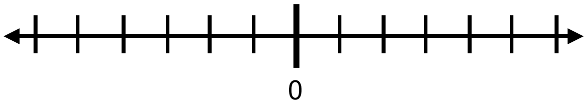
4. Using data from the table below, make an argument for whose bag of trail mix is more "peanut-y."

	Raisins	**Banana Chips**	**Peanuts**
Wrigley	12	10	18
Waverly	8	6	14

5. Circle all of the following expressions that are equivalent to 16x + 12y.

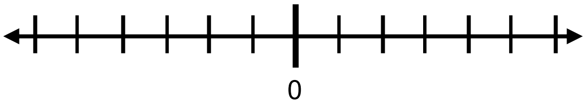
4(4x + 3y) 9x + 2(3x + 6y) 2(8x + 6y) 7x + 8y + 3y + 9x

6. Draw arrows to compute 4 + (-8).



7. Compute the following equation and then re-write it as an adding up equation and draw arrows to compute.

-3 – 6 = ___



For the given pairs of points on a line, find the distance between them.

8. 55 and 105	9. -50 and -80	10. 20 and -50

SKILL BUILDER 5

Draw a sketch using positive counters (+) and negative counters (–).

1. 3 groups of 2 positive counters	2. 2 groups of 3 positive counters	3. 4 groups of 3 negative counters

Write the change in temperature when placing the following in a liquid.

4. 3 groups of two hot pieces	5. 2 groups of 3 hot pieces	6. 4 groups of 3 cold nuggets

Compute. Refer to the counters/temperature scripts on pages 2 and 3 and draw diagrams as needed.

7. (3) • (-4)	8. (-3) • (2)	9. (-1) • (-6)
10. (-7) • (4)	11. (5) • (-2)	12. (-1) • (-7)
13. (9) • (-2)	14. (-5) • (-5)	15. (1) • (-8)
16. (-2)(4)	17. (5)(-3)	18. (-10)(-1)

SKILL BUILDER 6

Compute. Use the table on page 6 as needed.

1. (-24) ÷ (6) = _____ because _____ • 6 = -24	2. (25) ÷ (-5) = _____ because _____ • (-5) = 25	3. $\frac{-18}{-2}$ = _____ because _____ • (-2) = _____

Compute.

4. -12 • 4	5. 2 • (-13)	6. -5 • (-11)	7. -6 • 13
8. -30(-5)	9. 50(-3)	10. (20) ÷ (-5)	11. (-24) ÷ (-6)
12. -15 ÷ (-3)	13. $\frac{28}{-7}$	14. $\frac{-18}{-3}$	15. $\frac{-32}{8}$

16. Here are the melting points for some chemical compounds:

m-toluidine -30°C	biphenyl 69°C	1-hexanol -45°C

a. What numerical expression can be used to find the average melting point of the compounds?

b. What is the average melting point of the compounds? _____________

SKILL BUILDER 7

Compute.

1. $(-3) \bullet (-8)$	2. $7(-5)(-2)$	3. $-3 \bullet (-5 + 2)$
4. $-25 \div 5$	5. $-90 \div (-30)$	6. $56 \div (-8)$
7. $\frac{-81}{9}$	8. $-\left(\frac{77}{11}\right)$	9. $-\left(\frac{36}{-4}\right)$

Simplify each quotient.

10. $\frac{-9}{30}$	11. $\frac{-9}{-30}$	12. $-\left(\frac{9}{30}\right)$

13. Why is $\frac{-45}{9}$ equal to $-\left(\frac{45}{9}\right)$?

14. Why is $\frac{-2}{-8}$ not equal to $-\left(\frac{2}{8}\right)$?

Write each of the following as a quotient of integers.

15. 0	16. 0.31	17. $2\frac{1}{4}$

18. Why is $\frac{7}{0}$ not a rational number?

19. What does it mean to say that dividing by zero is undefined?

SKILL BUILDER 8

Justify each equation on the left with a property from the list on the right. You may use properties more than once.

	Equation	Property
1.	$0 = (-7)(0)$	
2.	$0 = 5 + (-5)$	
3.	$5 = (5)(1)$	
4.	$6 + (-3 + 3) = 6$	
5.	$(-6)(-5 + 1) = (-6)(-5) + (-6)(1)$	
6.	$(-4)(0) = (-4)(-1) + (-4)(1)$	
7.	$(-5 – 7)(-2) = (-5)(-2) – (7)(-2)$	

Properties

a. addition property of equality
b. additive identity property
c. additive inverse property
d. distributive property
e. multiplicative identity property
f. multiplication property of zero

Simplify.

8. $-2\frac{1}{4} + 1\frac{1}{3}$

9. $-2\frac{1}{3} + 3\frac{1}{2}$

10. $1\frac{1}{10} \bullet \left(-\frac{5}{8}\right)$

11. $-1\frac{1}{8} \div \left(-\frac{3}{4}\right)$

12. A newspaper reports these changes in the high temperature for the next 5 days: +3°, -9°, +2°, +1°, -12°. What is the average daily high temperature change?

FOCUS ON VOCABULARY

Use appropriate mathematical vocabulary to complete each sentence.

1. The result of multiplying numbers is called the ____________________ of the numbers.

2. The ____________________ of a positive number divided by a negative number is negative.

3. One rule for integer ____________________ states that the product of a negative number and a negative number is positive.

4. There is a(n) ______________________ relationship between multiplication and division. For example, if $3 \bullet 8 = 24$, then $24 \div 8 = 3$ and $24 \div 3 = 8$.

5. In problem 4, 8 is a(n) __________________ of 24.

6. A(n) _________________________ is a statement that is proposed to be true, but has not been proven to be true or false.

7. _______________________ is a form of reasoning in which the conclusion is supported by evidence, but it is not proved.

8. _______________________ is a form of reasoning where definitions, known facts, and accepted rules of logic are used to justify a conclusion.

9. A(n) ______________________ is a number that can be written in the form $\frac{m}{n}$, where m and n are integers, and $n \neq 0$.

10. A(n) _____________________ is a whole number or its opposite.

(For word hints, see the word bank and other vocabulary used in this packet.)

SELECTED RESPONSE

Show your work on a separate sheet of paper and choose the best answer(s).

1. Choose all the expressions that have a negative product.

A. (-7)(4) B. -4 • 7 C. -7(-4) D. 4 • 7

2. Choose all the expressions that have a product of 24.

A. (4)(6) B. (-6)(-4) C. 4 • 76 D. -4 • 6

3. Choose all the expressions that have a quotient of -4.

A. -12 ÷ (-3) B. 12 ÷ (-3) C. -12 ÷ 3 D. None of these

4. Which of the following expressions are not rational numbers?

A. $\frac{0}{3}$ B. $4\frac{1}{2}$ C. $\frac{5}{0}$ D. All of these are rational numbers.

5. Choose all the numbers that are equivalent to $\frac{2}{3}$.

A. $-\frac{2}{3}$ B. $\frac{-2}{-3}$ C. $\frac{2}{-3}$ D. $\frac{4}{6}$

6. Choose all the numbers that are equivalent to $-2\frac{3}{4}$.

A. $-\frac{10}{4}$ B. $-\frac{11}{4}$ C. $-\frac{5}{4}$ D. $\frac{-11}{-4}$

7. Choose all the expressions that represent a positive product.

A. (-2)•(-3)•4 B. (-5)(-6)(-2)

C. (3)(4)(-2) D. None of these expressions represents a positive product.

8. During a cold stretch in New York, the noon temperatures (in degrees Celsius) for 4 days were -3°, 2°, -8°, and 5°. What was the average temperature at noon over those 4 days?

A. -3° C B. -1° C C. 1° C D. 3° C

KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

5.1 Multiplication: Models

Compute. Refer to the counters / temperature change scripts on pages 2-3, and draw diagrams as needed.

1. (-4)(2)	2. (-4)(-2)

Use the words “positive” or “negative” to make each statement true.

3. The product of two negative integers is ____________________.

4. The product of a positive integer and a ________________ integer is negative.

5.2 Multiplication and Division: Patterns

Determine if each of the following statements are true or false. If a statement is false, rewrite it so it is true.

5. The quotient of a positive number and a negative number is negative.

6. The quotient of negative number and a negative number is negative.

Compute.

7. $\frac{-60}{-20}$	8. (320) ÷ (-20)

5.3 Reasoning About Products and Quotients

Compute, if possible.

9. $\frac{-6(7)}{3+2-5}$

HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.

Compute. Draw diagrams as needed.

1. (-3)(-9)

2. 8 • (-2)

Compute.

3. (-90) ÷ (-2)

4. $\frac{-45}{-15}$

5. Explain why $\frac{-12}{-6}$ is equivalent to $\frac{12}{6}$.

Parent (or Guardian) Signature_______________________________

This page is intentionally left blank.

COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT

6.NS 5*	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
7.NS.2a	Apply and extend previous understandings of multiplication and division ~~of fractions~~ to multiply and divide rational numbers. Understand that multiplication is extended ~~from fractions~~ to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
7.NS.2b	Apply and extend previous understandings of multiplication and division ~~of fractions~~ to multiply and divide rational numbers Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
7.NS.2c	Apply and extend previous understandings of multiplication and division ~~of fractions~~ to multiply and divide rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers.

*Review of content essential for success in 7th grade.

STANDARDS FOR MATHEMATICAL PRACTICE

MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP5	Use appropriate tools strategically.
MP7	Look for and make use of structure.
MP8	Look for and make use of repeated reasoning.

9 7 8 1 6 1 4 4 5 0 2 8 3