Center For
Mathematics
And Teaching

# Math Links

## SKILL BOOSTERS

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# GETTING STARTED WITH SKILL BOOSTERS

*MathLinks*: **Skill Boosters** are short, carefully sequenced daily skills practice, designed to help students fill gaps in knowledge without losing ground on current topics. The design of the program blends and adapts several recommendations confirmed to be effective by current research. Some of these include:

- Focus on whole numbers and rational numbers
- Explicit and systematic instruction
- Use of visual representations of mathematical ideas
- Short intervals (10 minutes in each session) to achieve fluency

To get started, take a close look at the Skill Booster sequences to familiarize yourself with the content. These topics represent the most common number sense skills needed for success in middle and high school mathematics.

- Whole Numbers
- Fraction Concepts
- Fraction Addition and Subtraction
- Fraction Multiplication and Division
- Decimal and Percent Concepts
- Integers
- Equations

Each topic includes pre- and post-assessments, daily problems, and a resource guide. If followed as designed, each topic cycle through 4 skills per week for 5 weeks. Use pre- and post-assessments to determine which Skill Booster topics are needed and to measure success.

Daily skill boosters should take no more than 10 minutes per day. Display the daily problems and ask students to complete problems on their own paper (such as in a journal) or on the template provided. Many teachers like to use skill boosters as class starters, or they may be used at any time.

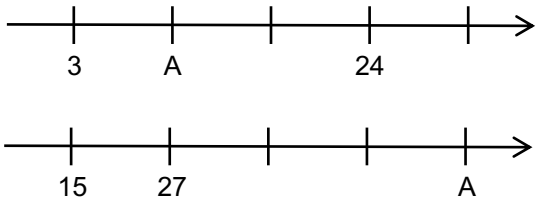
With a planned routine, students will have opportunities to fill gaps in a systematic way. We also encourage you to adapt the level of complexity of problems and time spent on skills to meet the needs of students. For example, if students are fairly fluent in whole number operations, those skills may be introduced more rapidly, reviewed within another sequence, or omitted entirely. Also, it may not be necessary for students to draw diagrams once they understand the meaning of a procedure (such as fraction addition).

With consistency and effort, we are confident that students will experience more computational fluency, which will lead to more success in mathematics!

## SKILL BOOSTERS

# WHOLE NUMBERS

# SKILL BOOSTERS: WHOLE NUMBERS

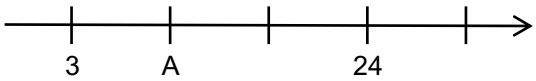
| Description | Problem Prompts / Examples | Resource Guide | Pre/Post Assessment Problems |
|---|---|---|---|
| Notation | Write 4 times 8 in three different ways<br>Write 24 / 6 in 3 different ways | Page 1 | 1 |
| Number facts | Compute: $7 \times 6$ | Page 3 | 3 |
| Multiply by powers of 10 ($10^n$) | Compute: $3 \times 10$, $3 \times 100$ | Page 1 | 5 |
| Whole number addition and subtraction | Compute: 48 + 63 | | 7 |
| Identify factors of numbers | What are the factors of 18? | Page 6, 7 | 9 |
| Find GCF and LCM | What is the GCF of 6 and 8?<br>What is the LCM of 6 and 8? | Pages 8, 9 | 2 |
| Whole number multiplication | Compute: $8 \times 315$ | Page 3 | 4 |
| Whole number division | Compute: $1024 \div 8$ | Pages 4, 5 | 6 |
| Order of operations | Compute: 5 + 6(3) | Pages 10, 11 | 8 |
| Locate numbers on a line (whole) | Find A:<br>  | | 10 |

Skills rotation: Each week, students practice four of the skills listed above daily.

| 5-week plan | A | B | C | D |
|---|---|---|---|---|
| **Week 1** | Notation | Number facts | Multiply by $10^n$ | Whole number addition and subtraction |
| **Week 2** | Number facts | Identify factors | Locate whole numbers on a number line | Whole number multiplication |
| **Week 3** | Notation | Multiply by $10^n$ | Identify factors | Whole number division |
| **Week 4** | Whole number addition and subtraction | Find LCM and GCF | Whole number multiplication | Order of operations |
| **Week 5** | Find LCM and GCF | Whole number division | Order of operations | Locate whole numbers on a number line |

# SKILL BOOSTERS: WHOLE NUMBERS
# PRE ASSESSMENT

Answer Key

| | |
|---|---|
| 1. Write 4 times 8 in three different ways.<br><br>4 × 8, 4(8), 4 • 8 | 2a. What is the GCF of 6 and 8?<br><br>2<br><br>2b. What is the LCM of 6 and 8?<br><br>24 |
| 3a. Compute: 7 × 6<br><br>42<br><br>3b. Compute: 9 × 8<br><br>72 | 4. Compute: 8 × 315<br><br>2,520 |
| 5a. Compute: 3 × 10<br><br>30<br><br>5b. Compute: 3 × 100<br><br>300 | 6. Compute: 1024 ÷ 8<br><br>128 |
| 7. Compute: 48 + 63<br><br>111 | 8a. Compute: 5 + 6(3)<br><br>23<br><br>8b. Compute: 8(6 – 4)<br><br>16 |
| 9. What are the factors of 18?<br><br>1, 2, 3, 6, 9, 18 | 10. Find A:<br><br>  <br><br><br>17 |

# SKILL BOOSTERS: WHOLE NUMBERS
# POST-ASSESSMENT

Answer Key

| | |
|---|---|
| 1. Write 12 times 6 in three different ways.<br><br>12 x 6 , 12(6), 12 • 6 | 2a. What is the GCF of 4 and 10?<br><br>2<br><br>2b. What is the LCM of 4 and 10?<br><br>20 |
| 3a. Compute: $8 \times 7$<br><br>56<br><br>3b. Compute: $6 \times 9$<br><br>54 | 4. Compute: $7 \times 428$<br><br>2,996 |
| 5a. Compute: $4 \times 100$<br><br>400<br><br>5b. Compute: $4 \times 1{,}000$<br><br>4,000 | 6. Compute: $1{,}362 \div 6$<br><br>227 |
| 7. Compute: 89 + 36<br><br>125 | 8a. Compute: $100 - 6(3^2)$<br><br>76<br><br>8b. Compute: 3 + 3(5)<br><br>18 |
| 9. What are the factors of 24?<br><br>1, 2, 3, 4, 6, 8, 12, 24 | 10. Find A:<br><br> <br><br><br>44 |

# SKILL BOOSTERS: WHOLE NUMBERS
# WEEK 1

Answer Key

## DAY 1

A. Write 4 times 8 in three different ways.

4 • 2 4(8) 4 × 8

B. Compute: 7 × 6

42

C. Compute: 3 × 10 30
Compute: 3 × 100 300

D. Compute in two different ways.
48 + 63

111

## DAY 2

A. Write 12(3) in three different ways.

12 • 3 12 times 3 12 × 3

B. Compute: 9 × 5

45

C. Compute: 24 × 100 2,400
Compute: 24 × 1,000 24,000

D. Compute in two different ways.
100 – 86

14

## DAY 3

A. Write 12 divided by 3 in three different ways.

$\frac{12}{3}$ 12 ÷ 3 $3\overline{)12}$

B. Compute: 49 ÷ 7

7

C. Compute: 8 × 100 800
Compute: 8 × 10 80

D. Compute in two different ways.
505 – 60

445

## DAY 4

A. Write $\frac{28}{4}$ in three different ways.

28 divided by 4 28 ÷ 4 $4\overline{)28}$

B. Compute: 48 ÷ 8

6

C. Compute: 10 × 100 1,000
Compute: 10 × 1,000 10,000

D. Compute in two different ways.
300 – 82 + 24

242

# SKILL BOOSTERS: WHOLE NUMBERS
# WEEK 2

Answer Key

## DAY 1

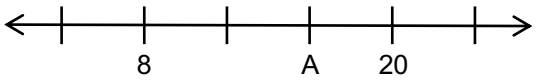
A. Compute: $6 \times 4$

24

B. What are the factors of 12?

1, 2, 3, 4, 6, 12

C. Find A:




16

D. Compute using 2 strategies:
$2 \times 308$

616

## DAY 2

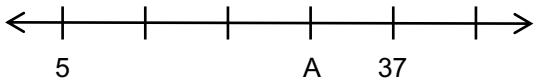
A. Compute: $8 \times 7$

56

B. What are the factors of 15?

1, 3, 5, 15

C. Find A:




29

D. Compute using 2 strategies:
$9 \times 136$

1,224

## DAY 3

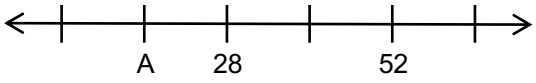
A. Compute: $42 \div 6$

7

B. What are the factors of 36?

1, 2, 3, 4, 6, 9, 12, 18, 36

C. Find A:




16

D. Compute using 2 strategies:
$5 \times 212$

1,060

## DAY 4

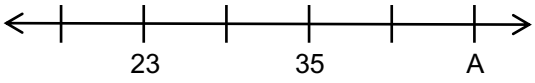
A. Compute: $45 \div 9$

5

B. What are the factors of 55?

1, 5, 11, 55

C. Find A:




47

D. Compute using 2 strategies:
$8 \times 167$

1,336

# SKILL BOOSTERS: WHOLE NUMBERS
# WEEK 3

Answer Key

## DAY 1

A. Write 7 times 6 in three different ways.

7 • 6 7(6) 7 × 6

B. Compute: 9 × 100 900
Compute: 9 × 1,000 9,000

C. What are the factors of 30?

1, 2, 3, 5, 6, 10, 15, 30

D. Compute using 2 strategies:
156 ÷ 3

52

## DAY 2

A. Write 11(12) in three different ways.

11 • 12 11 times 12 11 × 12

B. Compute: 11 × 10 110
Compute: 11 × 100 1,100

C. What are the factors of 18?

1, 2, 3, 6, 9, 18

D. Compute using 2 strategies:
216 ÷ 2

108

## DAY 3

A. Write 36 divided by 12 in three different ways.

$\frac{36}{12}$ 36 ÷ 12 $12\overline{)36}$

B. Compute: 20 × 100 2,000
Compute: 20 × 1,000 20,000

C. What are the factors of 24?

1, 2, 3, 4, 6, 8, 12, 24

D. Compute using 2 strategies:
1,092 ÷ 7

156

## DAY 4

A. Write 24 divided by 3 in three different ways.

$\frac{24}{3}$ 24 ÷ 3 $3\overline{)24}$

B. Compute: 6 × 1,000 6,000
Compute: 6 × 10,000 60,000

C. What are the factors of 21?

1, 3, 7, 21

D. Compute using 2 strategies:
895 ÷ 5

179

# SKILL BOOSTERS: WHOLE NUMBERS
# WEEK 4

Answer Key

## DAY 1

A. Compute: 57 + 32

89

B. Find the LCM of 4 and 8? 8
Find the GCF of 4 and 8? 4

C. Compute: 7 × 168

1,176

D. Compute: 13 + 2(4)

21

## DAY 2

A. Compute: 48 + 26

74

B. Find the LCM of 3 and 5? 15
Find the GCF of 3 and 5? 1

C. Compute: 6 × 389

2,334

D. Compute: $150 - 10^2$

50

## DAY 3

A. Compute: 62 – 13

49

B. Find the LCM of 6 and 9? 18
Find the GCF of 6 and 9? 3

C. Compute: 11 × 472

5,192

D. Compute: $8^2 - 4(5)$

44

## DAY 4

A. Compute: 452 – 24 + 16

444

B. Find the LCM of 12 and 24? 24
Find the GCF of 12 and 24? 12

C. Compute: 12 × 255

3,060

D. Compute: 25 + (233 – 65)

193

# SKILL BOOSTERS: WHOLE NUMBERS
# WEEK 5

Answer Key

## DAY 1

A. Find the LCM of 6 and 8? 24
Find the GCF of 6 and 8? 2

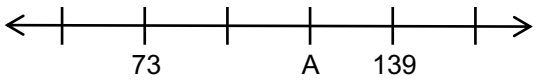
B. Compute: 504 ÷ 12

42

C. Compute: 72 ÷ 9 • 3

24

D. Find A.




117

## DAY 2

A. Find the LCM of 3 and 4? 12
Find the GCF of 3 and 4? 1

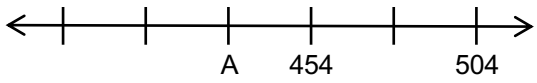
B. Compute: 225 ÷ 15

15

C. Compute: $28 + 7^2 - 4(5)$

57

D. Find A.




429

## DAY 3

A. Find the LCM of 12 and 20? 60
Find the GCF of 12 and 20? 4

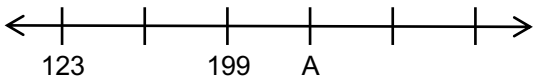
B. Compute: 768 ÷ 24

32

C. Compute: 38 + 25 + 5

43

D. Find A.




237

## DAY 4

A. Find the LCM of 5 and 30? 30
Find the GCF of 5 and 30? 5

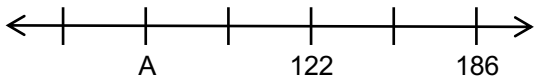
B. Compute: 1,650 ÷ 30

55

C. Compute: $52 - (3 + 4^2)$

33

D. Find A.

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# MATHEMATICAL SYMBOLS AND LANGUAGE

| Mathematical Symbols | | | |
|---|---|---|---|
| add | + | subtract | – |
| multiply | $\times$ • | divide | $\div$ / |
| is equal to | = | is not equal to | $\neq$ |
| is greater than | $>$ | is less than | $<$ |
| is greater than or equal to | $\geq$ | is less than or equal to | $\leq$ |
| is approximately equal to | $\approx$ | parentheses | ( ) |

## Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 $\quad 8 \times 4 \quad 8 \bullet 4 \quad (8)(4) \quad \begin{array}{r} 8 \\ \times\ 4 \end{array}$

In algebra, we generally avoid using the $\times$ for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol • for multiplication because it could be misinterpreted as a decimal point.

## Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $\quad 8 \div 4 \quad 4\overline{)8} \quad \frac{8}{4} \quad 8/4$

In algebra, the preferred way to show division is with fraction notation.

## Meanings for Exponents

In the expression $b^n$
- the number $b$ is the base
- the number $n$ is the exponent

$(\text{base})^{\text{exponent}}$

$b^n = \underbrace{b \bullet b \bullet b \ldots . b \bullet b \bullet b}$

multiplied by itself *n* times

$3^4 = 3 \bullet 3 \bullet 3 \bullet 3 = 81$

# WHOLE NUMBERS: MULTIPLICATION AND DIVISION

| Expanded Forms of Numbers | | | |
|---|---|---|---|
| Standard Form | Expanded Form #1 | Expanded Form #2 | Expanded Form #3 |
| 25 | 20 + 5 | 2(10) + 5(1) | $2(10^1) + 5(10^0)$ |
| 302 | 300 + 2 | 3(100) + 0(10) + 2(1) | $3(10^2) + 0(10^1) + 2(10^0)$ |

| Multiplication Strategies | | | | |
|---|---|---|---|---|
| **Skip Count** | **Double** | **Halve** | **Add On** | **Take Away** |
| 3<br>6<br>9<br>12<br>15 | $3 \times 7 = 21$,<br>so<br>$6 \times 7 = 42$<br>$7 \times 6 = 42$ | $6 \times 10 = 60$,<br>so<br>$6 \times 5 = 30$<br>$5 \times 6 = 30$ | $6 \times 3 = 18$<br>Think 19, 20, 21<br>so<br>$7 \times 3 = 21$<br>$3 \times 7 = 21$ | $10 \times 3 = 30$<br>$30 - 3 = 27$<br>so<br>$9 \times 3 = 27$<br>$3 \times 9 = 27$ |

**Multiplication Strategies (12 x 13)**

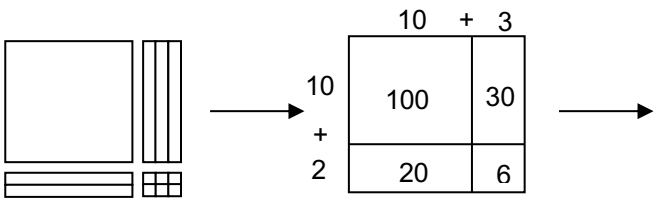
**Traditional Algorithm 1**

$$\begin{array}{r} 12 \\ \times\ 13 \\ \hline 36 \\ +\ 120 \\ \hline 156 \end{array}$$

**Traditional Algorithm 2**

$$\begin{array}{rl} 12 & \\ \times\ 13 & \\ \hline 6 & = 2 \times 3 \\ 30 & = 10 \times 3 \\ 20 & = 10 \times 2 \\ +\ 100 & = 10 \times 10 \\ \hline 156 & \end{array}$$

**Area Model**



| The Standard Division Algorithm | | |
|---|---|---|
| The standard division algorithm is an efficient process for dividing. It involves a cyclical process: divide, multiply, subtract, "bring down"… until the remainder is less than the divisor. | | |
| $14\overline{)963}$ | Determine where to start | Look at the divisor. Choose digits in the dividend so that the quotient using these digits is between 1 and 9. |
| $14\overline{)963}$ | Divide | How many 14s in 96? _____ Write this number above the 96. |
| $\begin{array}{r} \quad 6 \\ 14\overline{)963} \\ -84 \end{array}$ | Multiply | Find the product of 6 and 14. Write this below the 96. |
| $\begin{array}{r} \quad 6 \\ 14\overline{)963} \\ -84 \\ \hline 12 \end{array}$ | Subtract | Find the difference between 96 and 84. Write this below the 84. |
| $\begin{array}{r} \quad 6 \\ 14\overline{)963} \\ -84\downarrow \\ \hline 123 \end{array}$ | Bring down | Bring down the next digit. |
| $\begin{array}{r} \quad 68 \\ 14\overline{)963} \\ -84\downarrow \\ \hline 123 \\ -112 \\ \hline 11 \end{array}$ | Divide<br>Multiply<br>Subtract<br>Bring down (remainder) | Repeat the divide, multiply, subtract, bring down (if necessary) process until the remainder is less than the divisor. |

Some ways to represent the dividend, divisor, quotient, and remainder:

$\text{divisor}\overline{)\text{dividend}}$ $\quad \text{quotient} \quad \text{remainder}$

dividend = (divisor)(quotient) + remainder

$14\overline{)963} \quad 68 \text{ R } 11$

$14\overline{)963} \quad 68\ \frac{11}{14}$

963 = (14)(68) + 11

## A Chunking Division Procedure

This chunking division procedure keeps the dividend intact as we "close in" on the quotient. If you do not know all your multiplication facts, this procedure may be easier than the standard division algorithm because you subtract out groups of the divisor more flexibly, but still arrive at the correct quotient. If the largest amount possible is chosen to subtract at each step, this procedure is very efficient.

**Divide 761 highlighters into 3 boxes.**

Step 1. Rewrite the problem.

$3\overline{)761}$

Step 2**:** Make a toolkit of multiplication facts that may be useful for this problem.

| | | |
|---|---|---|
| $3 \times 1 = 3$ | $3 \times 10 = 30$ | $3 \times 100 = 300$ |
| $3 \times 2 = 6$ | $3 \times 20 = 60$ | $3 \times 200 = 600$ |
| $3 \times 3 = 9$ | $3 \times 30 = 90$ | $3 \times 300 = 900$ |
| $3 \times 4 = 12$ | $3 \times 40 = 120$ | $3 \times 400 = 1{,}200$ |

Step 3: Select a fact from the toolkit that is less than or equal to the dividend, and record.

| | |
|---|---|
| 761 | |
| − 600 | 200 |
| 161 | |

Repeat Step 3: Continue the routine until the remainder is less than the divisor.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 253 | R 2 | |
| 761 | | | 761 | | | 761 | | | |
| − 600 | 200 | → | − 600 | 200 | → | − 600 | 200 | | |
| 161 | | | 161 | | | 161 | | | |
| − 120 | 40 | | − 120 | 40 | | − 120 | 40 | | |
| 41 | | | 41 | | | 41 | | | |
| | | | − 30 | 10 | | − 30 | 10 | | |
| | | | 11 | | | 11 | | | |
| | | | | | | − 9 | | + 3 | |
| | | | | | | | | 253 | |

The last calculation shows that the quotient is (200 + 40 + 10 + 3) = 253, and the remainder is 2.