UNIT 1
TEACHER EDITION

Math Links
GRADE 6

BLUE Annotations for Preview

STATISTICS

Unit Planning Information (TE-UPI)
Numbered i, ii, iii, …

Annotated Answer Key, including Student Packet and Lesson Notes (TE-AK)
Numbered 0, 1, 2, …

Each MathLinks Unit is organized this way.

The start of TE-UPI	**COMMON CORE STATE STANDARDS**

STANDARDS FOR MATHEMATICAL CONTENT	
6.SP.A	**Develop understanding of statistical variability.**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
6.SP.B	**Summarize and describe distributions.**
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
6.SP.5	Summarize numerical data sets in relation to their context, such as by:
a.	reporting the number of observations.
b.	describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c.	giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d.	relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Common Core State Standards for the Unit are listed here and also at the end of the Student Packet.

Individual sets of standards are listed at the beginning of each lesson.

Standards for the entire grade level are in Program Information.

UNIT PLANNING

* Starred (*) resources can be accessed under Unit Resources on the Teacher Portal.

Unit Pacing* Up to 14 class hours Pacing detail (estimates)	1.0 1.1 1.2 1.3	Opening Problem: Beach Cleanup (< 1 hour) Introduction to Data Analysis (3 hours) More Measures of Center and Spread (3 hours) Data Displays (3 hours) Review (3 hours) Assessment (1 hour)
Unit Resources* Up to 3 class hours On the Teacher Portal	• Extra Problems • Essential Skills • Math Talks (Data, Dot) • Nonroutine Problems • Technology Activities	• Tasks (Choosing a Service Project) • Projects Conduct a Survey, Ages of Presidents) • Parent Support Letters

Assessment Options*
 	
See Teacher Portal Unit 1 for specific diagnostic and follow-up ideas.
 	
Details in Program Information
	• On the Teacher Portal
	  ✓ Unit Quizzes
	  ✓ Cumulative Tests
	  ✓ Tasks
	  ✓ Projects
	  The *MathLinks* Rubric
	  (with rubric-worthy problems)
		• In the Student Packet
		  ✓ Monitor Your Progress
		  ✓ Unit Reflection
		• In the Teacher Edition
		  ✓ References to Journals
		  ✓ Suggested problems for The *MathLinks* Rubric

Materials See Program Information for supplies for the year.	• Sticky notes [1.1] (1 per student) • Linking cubes or other small objects [1.2] (10-15 per student) or https://www.didax.com/math/virtual-manipulatives.html • Computers with internet access [Review] • General supplies (e.g., colored pencils, markers, rulers, tape, scissors, graph paper, calculators, chart paper)	
Slide Decks* On the Teacher Portal	S1.1a Name Scores S1.1b Statistical Questions S1.2 Name Scores Revisited S1.3a Three Data Displays S1.3b Interpreting Histograms S1.3c Interpreting Box Plots	
Reproducibles* At the end of TE-UPI	R1-1 Five-Number Summary Cards [1.1] (1 copy) R1-2 Measures of Center and Spread Cards [1.1] (1 copy) R1-3 Blank Strips [1.1] (1 per small group, optional) R1-4 These Are – These Aren't [1.1] (1 per small group or pair) R1-5 Waqueyzaquey Card Sort [1.2] (1 per pair or small group) R1-6 Mean and MAD Template [1.2] (1-2 per student) R1-7 Match and Compare Sorting Cards: Statistics [Review] (1 per pair)	
Prepare Ahead Use Activity Routines files in General Resources at the start of the year.	• Ask early finishers to cut up R1-1, R1-2, R1-3, R1-4, R1-5, and R1-7 [1.1, 1.2, Review] See Activity Routines in Program Information for directions for the *MathLinks* Rubric, Poster Problems, Match and Compare Sorts, Why Doesn't It Belong?, and Computational Fluency Challenge [1.2, Review]	
Other Resources on the Teacher Portal Watch Getting Started Videos in General Resources at the start of the year.	• Getting Started Videos and Resources: General Resources • Skill Boosters: Teacher Access page (whole numbers, fractions) Puzzles / Games: Teacher Access page (SMASH Game 1, Pattern Grids Unlocked)	

PLANNING FOR DIFFERENT USERS

Student Packet (SP)

Unit 1 component options for those who support students:

For teachers	• Teacher Edition (this document) • Teacher Portal (Unit Resources, General Resources) • Program Information
For substitutes	• SP (Practice 1 – 7 may be completed independently any time after instruction; Spiral Review; Vocabulary Review) • Extra Problems • *MathLinks* Puzzles / Games
For parents	• Resource Guide • Parent Letter (English and Spanish)

Print copy and on the Portal

Created by Carole Greenes for MathLinks users; on the Portal; not linked to any course

Unit 1 component options to use with all students (all available on the Teacher Portal):

• SP (Word Bank, ⌷ Activity Routines, ⌷ self-monitoring, j⌷ • Student Packet T⌷ • Extra Problems (⌷ • Essential Skills (j⌷	• Math Talks (whole-class discourse) • Nonroutine Problems (enrichment) • Tasks (multi-part problems) • Projects (authentic multi-day experiences) • Technology Activities (variety) • *MathLinks* Puzzles / Games (fun challenges)

In addition to unit-by-unit Parent Letters, the public website also has an introductory letter.

Unit 1 component options for particular subgroups of students:

For English learners see pages vii-viii for specific strategies	• SP Text File for Translation • SP features for language development (Word Bank, Vocabulary Review, consistent structure for reading and writing, and grouping opportunities for speaking and listening) • SP Activity Routines for language development (rubric-worthy problems with the *MathLinks* Rubric, Match and Compare Sort, Why Doesn't It Belong?, Poster Problems) • Math Talks for speaking and listening
For struggling learners see pages vii-viii for specific strategies	• SP features for math confidence (Getting Started, Review including Spiral Review, Word Bank, Vocabulary Review, consistent structure, grouping options) • SP Activity Routines for math confidence (rubric-worthy p⌷ Rubric, Match and Compare Sort, Why Doesn't It Belong⌷ Computational Fluency Challenge) • Essential Skills for just-in-time intervention • Extra Problems (by lesson) for practice, review, or assessment • Skill Boosters (Whole Numbers, Fraction Concepts, Fraction Addition and Subtraction)
For enrichment and advanced learners	• SP features for enrichment (see pg xi for specific options) • SP Activity Routines for enrichment (rubric-worthy problems with the *MathLinks* Rubric, Match and Compare Sort, Why Doesn't It Belong?) • Nonroutine Problems (including problems from the Math Olympiad) • Technology Activities for variety • Projects

Use a translation app

A skills practice routine; on the Portal; not linked to any course

Activity Routines recur throughout a course. Use the introductory activities in General Resources first.

MATH BACKGROUND

| **"The Quartile" or "In the Quartile"** |
|---|

The word "quartile" is used in statistics in two different ways. Most often, it is used to denote numbers that separate an ordered data set into four equal parts. In the sibling data set {2, 2, 2, 2, 3, 3, 4, 5, 5, 7, 11}, Q_3 (or the third quartile) is 5.

The word "quartile" can also refer to a set of values, namely, one of those four equal parts. In the data set above, the fourth quartile is the set {5, 7, 11}.

Thus, "the first quartile is 2," but "the value 2 lies both in the first quartile and in the second quartile." This ambiguous use of terms occurs often in mathematics. For instance, the word "circle" usually refers to the boundary of a disk, but it can also refer to the entire disk.

The Tale is in the Tail

A data set that is not symmetric is said to be <u>skewed</u>. One way a data set can be skewed is to have a tail, that is, a higher concentration of data at one end of the distribution than the other, as indicated in the figures below. We say the data is <u>skewed to the right</u> if the tail of the distribution goes to the right. And we say the data is <u>skewed to the left</u> if the tail of the distribution goes to the left. The tail may provide valuable insight into the data set when interpreted in context.

tail (skewed to the left)	tail (skewed to the right)	symmetric
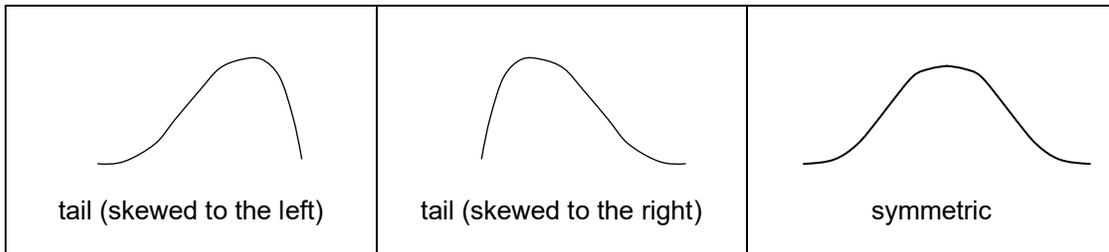
Using Data to Understand Our World

A key to understanding the world we live in is to collect data: temperature, wind velocity, rainfall, water levels, animal populations, cases of malaria, and so on. The data have a story to tell. It is the job of mathematics and statistics to read the story. We need mathematics and statistics to tell us what the trends are and with what certainty. Once we've read the story, it is our job to take steps to make the world a better place to live in.

> Math Background includes adult-level information and explanations written by our PhD mathematicians.

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)	
Here is an abbreviated version of the SMPs and some ways they are applied in this unit.	

<table>
<tr>
<td rowspan="1">SMP1</td>
<td>Make sense of problems and persevere in solving them.
Understand a problem and look for entry points
Consider simpler or analogous problems
Monitor progress and alter solution course as needed
Make connections between multiple representations
Check answers with a different method</td>
<td>[1.0, 1.2] Students interpret data before and after they have statistical tools.

[1.0, 1.2] Beach Cleanup and Practice 2 provides students the opportunity to explore statistical information regarding a beach cleanup project and determine if commitments are being met using statistical tools.</td>
</tr>
<tr>
<td>SMP2</td>
<td>Reason abstractly and quantitatively.
Use numbers and quantities flexibly in computations
Attend to the meaning of quantities
Decontextualize a problem using symbols, manipulate them, and then interpret based on the context</td>
<td>[All Lessons] Students analyze data using quantitative methods and use reasoning to answer statistical questions.</td>
</tr>
<tr>
<td>SMP3</td>
<td>Construct viable arguments and critique the reasoning of others.
Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws
Make and test conjectures based on evidence
Analyze situations by breaking them into cases
Understand and analyze the approaches of others</td>
<td>[All Lessons] Students defend conclusions drawn from statistical measurements and data displays using analytical reasoning.

[1.3] Students analyze the work and statements of two students, who make and analyze data displays.</td>
</tr>
<tr>
<td>SMP4</td>
<td>Model with mathematics.
Attach meaningful mathematics to everyday problems and questions of interest
Make reasonable assumptions and approximations to simplify a situation
Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships
Interpret results and draw conclusions in the context of the situation</td>
<td>[All Lessons] Students use statistical measures of center and spread in real-world applications and determine which of these are appropriate for describing particular data sets. Students analyze graphical models of statistical displays to answer questions that encourage the contextualizing of numerical information. Students generate their own statistical displays to describe real-world data.</td>
</tr>
</table>

Abbreviated descriptions of the Standards for Mathematical Practice appear in every unit.

Specific examples from the Student Packet for this unit.

	Applying Standards for Mathematical Practice (SMP) Continued	
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	[1.1] Students use objects to create physical meaning for the mean and strips to create physical meaning for the five-number summary. [1.2, 1.3, Review] Discuss with students why it is important to use a calculator for finding statistical measures for larger data sets. Students may want to try an online MAD calculator too. Paper and pencil calculations can be tedious and may get in the way of important focal points of the lessons.
SMP6	**Attend to precision.** • Calculate accurately and efficiently • Explain thinking using mathematical vocabulary • Use symbols appropriately • Specify units of measure	[All Lessons] Students record mathematics vocabulary as it is introduced in lessons. They use precise language in writing and exercises. Precise definitions are located in the Student Resources section in the back of the unit. [1.2, Review] Students use calculators or online statistical tools (like a MAD calculator) when appropriate. [1.3] Students label and scale data displays appropriately.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler object	[1.2] When finding the mean with manipulatives, students connect the leveling of stacks of cubes to the division procedure.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations	

Strategies to Support Different Learners	
Classrooms typically include students with different learning styles and needs. Here are some specific ways that *MathLinks* supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See Universal Design for Learning Information for more details.	
General Examples	***MathLinks* Examples**

Know your Learner

- ✓ Understand student attributes that support or interfere with learning
- ✓ Determine preferred learning and interaction styles
- ✓ Assess student knowledge of prerequisite mathematics content
- ✓ Check for understanding continuously
- ✓ Provide differentiation opportunities for intervention to reach more learners
- ✓ Encourage students to write about their attitudes and feelings towards math
- ✓ Use contexts that link to students' cultures*

Built into the *MathLinks* Design:
SP: Getting Started, Spiral Review, Monitor Your Progress, Unit Reflection
TE: References to Journals
UR: Extra Problems, Essential Skills, Projects
OR: Skill Boosters, Assessment Options

[1.1, 1.2, 1.3] Start the year encouraging to students to write about their attitudes towards math. Some journal prompt ideas are identified in Activity Routines.

[1.2, 1.3] Pre-assess students' division computation fluency when they are required to compute the mean.

[All] Observe students' preferred learning styles and language proficiency during direct instruction, activities, group work, discussions, and individual work.

Many strategies are consistently in components. They are built into the MathLinks design

Increase Academic Language through Mathematics

- ✓ Provide opportunities for students to read, write, speak, and listen
- ✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples
- ✓ Use strategically organized groups that attend to language needs*
- ✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development*
- ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*

A "" indicates a strategy for English learners.*

Built into the *MathLinks* Design:
SP: Word Bank, Vocabulary Review, Student Resources
TE: Grouping suggestions, References to Journals, Suggested problems for **The *MathLinks* Rubric**
UR: Math Talks
OR: Critique student work on Slide Decks

[1.1] Distribute the reproducible **R1-1 Five-Number Summary Cards** and **R1-2 Measures of Center and Spread Cards** as part of the name score line up to help all students understand the meaning of these words.*

[1.2] Discuss the meaning of homophones "tale" and "tail." Then use the phrase "the tale is in the tail" to help students remember if data is skewed left or right.*

[Review] Use **Match and Compare Sort** to build academic vocabulary, which is referenced throughout the unit.*

[1.1, 1.2, 1.3] Practice problems noted as journal prompts give students opportunities to write about their thinking, and they make good exit slips.

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (PR), Other Resource (OR)

These General Examples appear in every unit.

Specific examples from the Student Packet for this unit

Strategies to Support Different Learners (Continued)	
General Examples	***MathLinks* Examples**

<table>
<tr>
<td rowspan="2">Increase Comprehensible Input</td>
<td>

✓ Link concepts to past learning

✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding

✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use

✓ Use graphic organizers to help students record information and data, see patterns, and generalize them

✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections

✓ Strategically sequence and scaffold to make mathematics accessible

✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*

</td>
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Built into the *MathLinks* Design:
SP: Structured workspace
TE: Slide Deck Alternatives, Reproducibles, Materials
OR: Slide Decks

[1.1] The student name score line up activity engages students with data through a "total physical response."

[1.1] Using strips to find the five-number summary (see Teaching Tips) helps students visually interpret this concept.*

[1.2] Finding the mean by sharing objects provides a hands-on strategy for understanding this concept.*

[All] Students perform computations, create data displays, and explain the meaning of their analysis of statistical questions.

</td>
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</table>

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<tr>
<td>Promote Student Interaction</td>
<td>

✓ Use flexible group configurations that support content objectives

✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems)

✓ Encourage elaborate responses through questioning

✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language*

✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)*

</td>
<td>

Built into the *MathLinks* Design:
SP: Lesson and Review activities
TE: References for Journals, Suggested problems for **The *MathLinks* Rubric**
UR: Math Talks, various games and puzzles
OR: Slide Decks, Activity Routines

[1.1, 1.2, Review] To promote discussion in a safe environment, organize students into pairs or small groups for the reproducible card cut-up activities (**R1-4 These Are-These Aren't**, **R1-5 Waquezaquey Card Sort**, **R1-7 Match and Compare Sorting Cards: Statistics**) Students will gain confidence as they explain their reasoning first to each other, then to the larger group.

[All] Encourage students to explore a statistical question of interest to them.
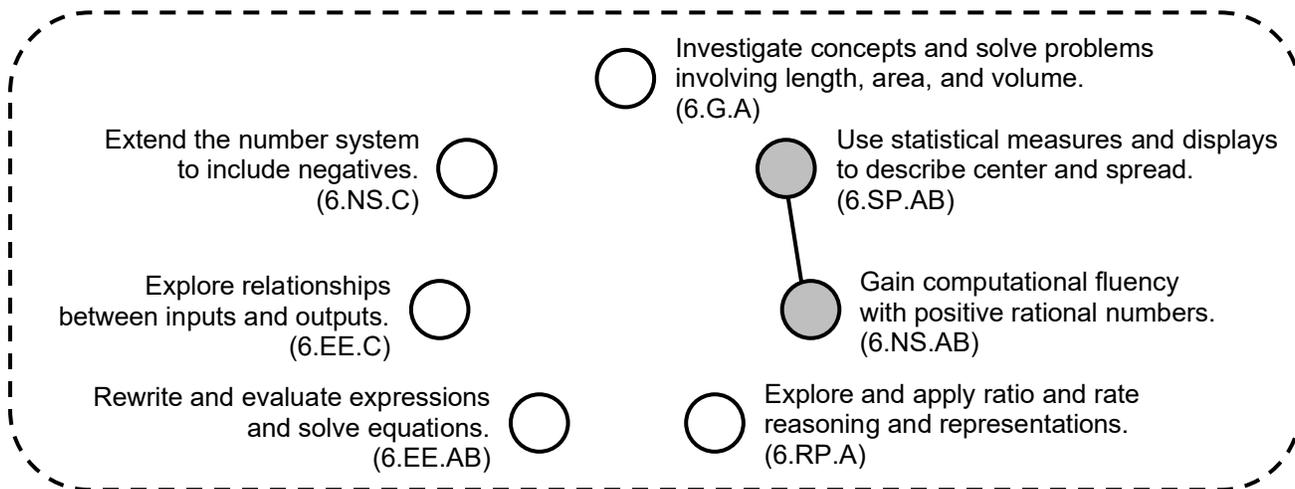
</td>
</tr>
</table>

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (PR), Other Resources (OR)

> Strategies to Support Different Learners combine the Universal Design for Learning (UDL) with other research-based strategies that are proven successful for a wide range of learners, especially those with special needs and English learners. See program Information for more details and a bibliography of cited references and resources for inspiration.

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 6 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Unit 1 and their connections to each other.

Investigate concepts and solve problems involving length, area, and volume. (6.G.A)

Extend the number system to include negatives. (6.NS.C)

Use statistical measures and displays to describe center and spread. (6.SP.AB)

Explore relationships between inputs and outputs. (6.EE.C)

Gain computational fluency with positive rational numbers. (6.NS.AB)

Rewrite and evaluate expressions and solve equations. (6.EE.AB)

Explore and apply ratio and rate reasoning and representations. (6.RP.A)

These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	**What's Ahead**
• Represent and interpret data, especially line plots (4.MD.B, 5.MD.B) • Add, subtract, multiply and divide whole numbers and decimals to hundredths (5.NBT.B) Connections made to: • work from previous courses or from earlier in the grade (above) • work later in grade or to future courses (to the right)	• Use random sampling to draw inferences about population(s) (7.SP.A) • Investigate bivariate data (8.SP.A) • Interpret categorical and quantitative data (HS) • Make inferences and justify conclusions about statistical experiments, surveys, and observational studies (HS) • Use statistics as a tool when mathematical modeling (HS)

The "Big Idea wheel" helps teachers and students see connections among mathematical ideas within a unit. Big Ideas also ensure a cohesive and efficient design program. The Big Idea wheel also appears on the Reflection page of every Student Packet. The *MathLinks* version was inspired by the work of Randy Charles and Jo Boaler (see the References section of Program Information, Portal version only). It consists of Domains from the Common Core State Standards.

Developing Language Skills through *MathLinks*

Language (reading. writing, speaking, and listening) helps students communicate math ideas and understand concepts. Here are examples for Unit 1.

Language Objectives

Students will:

(Lesson 1) Discuss orally the important vocabulary that is introduced in context during the Name Score activity, in addition to the physical/visual component, before students ever see the formal definitions.

(Lesson 2) Discuss and write about new words in a mathematical context that have dual meanings (see New Language Journal below). By now, students have experienced several that they may need extra support unpacking (e.g., mean, median, mode, range, tail).

(Lesson 3) Interpret data displays orally and in writing.

(Review) Contribute ideas in conversation during discussion to the solutions of *all* activities in this section prior to writing them.

Group Discussions to Promote Reading, Listening, and Speaking (2+)

Critique reasoning situations appear on slide decks for discussion (S1.3b, S1.3c) and writing (Interpreting Box Plots pg 17). Use a partner strategy such as "turn and talk" to encourage students to respectfully critique the reasoning of the fictional students.

For Statistical Questions (pg 5), encourage individuals to read the cards before pairs/trios discuss whether they are statistical questions or not.

For Three Data Displays (pg 13), pairs/trios should discuss how they want to scale the two number lines; and for the histogram how to scale the vertical axis and partition the horizontal sections. Encourage respectful critiquing of ideas, and stress that there may not necessarily be only one correct way.

For all Review section activities, group students in a way that makes them feel safe in expressing their ideas orally before writing their answers in the packet.

Journal Ideas to Promote Writing

(Explaining Concepts) See pgs 4, 9, 14 for specific content Journal prompts.

(New Language) Choose at least one entry from the Word Bank that has a conversational English meaning and a mathematical meaning. Describe whether these two meanings have similarities or not.

(Language in the Real World) Explain a situation in the real world where someone might use data and statistics studied in this unit. Explain why you think it might be important.

Students May Wonder…

- Darrell Huff was a journalist, not a statistician. He wrote a book in 1954 called "How to Lie with Statistics" that has been updated and is still popular today. Students may want to consider what this means, find examples, or search the internet for some of Darrell Huff's quotes.

- Encourage students to research apps on their phone, a spreadsheet, or online, to compute statistics. For example, they may want to search for a "MAD calculator."

Enrichment and Challenges for Advanced Learners

MathLinks: Grade 6 materials provide multiple opportunities for advanced students to investigate grade-level mathematics at a higher level of complexity without doing more work than their peers.

Within this Student Packet, here is a page with accessible entry points and opportunities for extensions (low floors / high ceilings).

- Beach Cleanup (pgs 1, 8 problem 5)

Every student does not need to do every problem in a Student Packet. Challenge those who are ready with these pages. Others might do unfinished work, Spiral Review, or more practice problems as needed.

- Practice 4: Extend Your Thinking (pg 11)
- Mean Absolute Deviation (pg 9) with Practice 3 (pg 10, problem 4) . As an added extension, encourage advanced learners to research the "mean absolute deviation" statistic on the internet.

Consider speeding up instruction, and skipping some Practice and Spiral Review. See also **Planning for Different Users** (TE, pg iii) and "Examples of Strategies for Enrichment and Advanced Learners" (Program Information → Universal Design for Learning) for more ideas.

6th Grade Fluency Requirements

In Grade 6, the Common Core State Standards (K-6) sets expectations for fluency in computation (e.g., "Fluently divide multi-digit numbers" [6.NS.2] and "Fluently add, subtract, multiply, and divide multi-digit decimals" [6.NS.3] using the standard algorithm).

In previous grades, students solve whole number and decimal problems using a progression of sense-making methods that build towards the standard algorithm in grade 6. Some Grade 5 examples:

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [5.NBT.6]

- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [5.NBT.7]

The word fluent is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems.

MathLinks: Grade 6 supports students in attaining computational fluency by embedding problems within lessons, and via routines strategically placed throughout the program. These include **Fluency Challenges** in the Spiral Review section and **Four in a Row** games in **Nonroutine Problems**.

Adapted from:
The University of Arizona. 2011-2013. Progressions Documents for the Common Core Math Standards. Tucson AZ: Institute for Mathematics and Education, University of Arizona. Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking.

Fluency is always important, and these are actual requirements in 6th grade.

The Three Components of Rigor in *MathLinks*

A group of educators at Rutgers University led by Dr. Roberta Shorr (Edutopia, October 2009) conducted a seven-year study on problem solving with minority and low-income students in low performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent).

The Common Core State Standards for Mathematics describes rigor as the pursuit of conceptual understanding, procedural skills and fluency, and applications with equal intensity. *MathLinks* courses embrace that description.

- Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It's a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented. This is commonly done throughout lessons in every unit, oftentimes with the help of teacher Lesson Notes and slide decks (or Slide Deck Alternatives). Extra conceptual understanding opportunities are found in each unit's accompanying Unit Resources files (located on the Teacher Portal). The Math Talks and Nonroutine Problems sections are filled with much to choose from.

- Problem solving is another driver within *MathLinks* courses. All units begin with an Opening Problem. In many cases, students require more instruction throughout the unit before they are fully prepared to bring the problem to its conclusion. More "meaty" problems exist in many units. Extra problem-solving opportunities are found in each unit's accompanying Unit Resources files (located on the teacher portal). View the Tasks and Projects sections for options.

- Practice pages are found throughout each unit, as well as Spiral Review in the Review section. Extra skills practice opportunities exist on the teacher portal in these formats:
  - Skill Boosters offer structured practice in a handful of frequently requested topic areas.
  - Unit Resources has an accompanying Extra Problems file from which teachers can choose exercises to use for more skills practice or assessment.
  - Unit Resources has an accompanying Essential Skills section that includes topics from previous grades to help students who might not be ready to fully engage in the given unit.

Young, Bernice. "Kids Master Mathematics When They're Challenged But Supported." Edutopia, 9 Sept. 2009, www.edutopia.org/math-underachieving-mathnext-rutgers-newark.

Using Strips to Find the Five-Number Summary

Suppose these numbers represent the number of siblings for 13 different students.

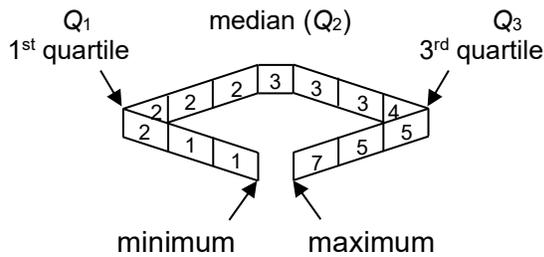<div align="center">3,   4,   5,   2,   2,   3,   3,   2,   2,   5,   7,   1,   1</div>

Enter these numbers, in numerical order, on a blank strip.

1	1	2	2	2	2	3	3	3	4	5	5	7

Fold the strip as shown below to locate the minimum, 1st quartile (Q_1), median (Q_2), 3rd quartile (Q_3), and maximum.

(Odd Number of Data Values)

	Class Siblings
Minimum	1
1st Quartile (Q_1)	2
Median (Q_2)	3
3rd Quartile (Q_3)	4.5
Maximum	7

Teaching Tips later in the TE-UPI are about specific instructional strategies. For this one above, note that the strips are on an upcoming Reproducible page, and there is also an electronic version of them in the Unit Resources for the Unit 1 TE.

REPRODUCIBLES

R1-1 FIVE-NUMBER SUMMARY CARDS

Reproducibles are typically for games, card sorts, templates, and manipulatives. They are listed on the TE-UPI Unit Planning page (always page ii) with details of where they are used and how many copies are needed. There is also a reminder on the TE-AK page where they are needed, though sometimes they are listed as optional.

MINIMUM VALUE	**FIRST QUARTILE (Q_1)**
SECOND QUARTILE (Q_2)	**THIRD QUARTILE (Q_3)**
MAXIMUM VALUE	**RANGE**

R1-2 MEASURES OF CENTER AND SPREAD CARDS

MEAN	**MEDIAN**
MODE	**MODE**
POTENTIAL OUTLIER	**MODE**

R1-3 BLANK STRIPS

R1-4 THESE ARE–THESE AREN'T

A. Is your first name score higher than your last name score?	L. What is the capitol of California?
B. What is the favorite sports team of middle school students?	M. Do you like peanut butter?
C. What grade did you get on the last test?	N. What is the favorite subject of your classmates?
D. What is the weight of your dog?	O. Which tends to be the higher score – first names or last names?
E. Who likes apple juice more: 1st graders or 7th graders?	P. How many presidents were under 50 when inaugurated?
F. What time do you go to bed on a school night?	Q. What was the average score on the last test?
G. Did the Dallas football team make the playoffs last year?	R. What is the most expensive item on the menu?
H. What is your name score?	S. What do female Great Danes usually weigh?
I. What is the typical name score in our class?	T. On average, how old are the pet dogs in our class?
K. How many days are in March?	U. What is the average shoe size of a 6th grader?

R1-5 WAQUEYZAQUEY CARD SORT

A. I think Waqueyzaquey's name score will make the <u>mean</u> of the class scores…	I. … decrease a lot.
B. I think Waqueyzaquey's name score will make the <u>median</u> of the class scores…	II. …decrease a little.
C. I think Waqueyzaquey's name score will make the <u>mode</u> of the class scores…	III. …remain unchanged.
D. I think Waqueyzaquey's name score will make the <u>range</u> of the class scores…	IV. … increase a little
E. I think Waqueyzaquey's name score will make the <u>interquartile range</u> of the class scores…	V. …increase a lot.

R1-6 MEAN AND MAD TEMPLATE

Use for up to 10 data points. For large data sets consider using an online statistics calculator.

number of data points	**MEASURE OF CENTER** List the data points.	**MEASURE OF SPREAD** Find the positive difference from the mean.
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
	Sum = Average (MEAN) =	Sum = Average (MAD) =

Number of observations (*n*) = _____   Mean = _____   MAD = _____

- -

number of data points	**MEASURE OF CENTER** List the data points.	**MEASURE OF SPREAD** Find the positive difference from the mean.
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
	Sum = Average (MEAN) =	Sum = Average (MAD) =

Number of observations (*n*) = _____   Mean = _____   MAD = _____

R1-7 MATCH AND COMPARE SORTING CARDS: STATISTICS

I △ **MEAN**	I ○ **MEDIAN**
II △ **MEAN ABSOLUTE DEVIATION**	II ○ **INTERQUARTILE RANGE**
III △ **DOT PLOT**	III ○ **BOX PLOT**
IV △ **MEASURES OF CENTER**	IV ○ **MEASURES OF SPREAD**
A △ ✓ A single number that describes the middle of a data set ✓ Includes mean, median, or mode	A ○ ✓ Data display of the five-number summary ✓ The "whiskers" represent values in the first and fourth quartiles.
B △ ✓ Average of a data set ✓ Found by adding data values and dividing by the number of observations	B ○ ✓ Measures that describe the spread of a data set ✓ Includes IQR and MAD
C △ ✓ A data display that shows values and frequencies with dots or x's ✓ Display is typically above a number line	C ○ ✓ Middle data value when data values are organized from least to greatest ✓ Considered a reliable way to report the center of a data set
D △ ✓ Measure of spread associated with the mean ✓ Average of the distances of the values in a data set from the mean	D ○ ✓ Measure of variability associated with the median ✓ Half of the data is captured in this range

Statistics

Which tool is best for math? The multi-pliers.

We hope you like our little math jokes more than "this page was left blank intentionally."

The end of
TE-UPI

Name_____   Period _____   Date_____

UNIT 1
ANSWER KEY

Math Links
GRADE 6

STATISTICS

Materials   Grouping   Reproducibles   Slide Deck   Journal Idea

Look for these icons at the bottom of many TE-AK pages for instructional suggestions.

Parent (or Guardian) signature _____

Parent Support letters are available on the Teacher Portal AND on the public website for those who help your students at home.

MathLinks: Grade 6 (2nd ed.) ©CMAT
Unit 1: Teacher Edition

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

five-number summary interquartile range (IQR) *When a vocabulary word first comes up in context, take the time to support students in writing something that is meaningful to them, whether it's an explanation of the vocabulary in their own words, an example, and/or a picture.*	mean
mean absolute deviation (MAD) *This is the first of many annotations you'll see in red comic sans. This one is meant as a timely teaching note.*	median
mode	outlier
range (of a data set)	statistical questions

OPENING PROBLEM: BEACH CLEANUP

[SMP 1, 2, 3, 4]

Revisit this problem after students learn more about data and averages.

Students at Moffett Middle School ~~service for 120 minutes, on avera~~

Everette reads that the beaches ar~~bottles, so he chooses to serve by~~

Here is Everette's beach cleanup l~~break.~~

> Every unit begins with an opening problem to motivate learning and pique interest. Some are wrapped up within a class hour. Many require more concepts and skills than students have at the beginning of a unit, so they are revisited later. When to revisit is always stated directly in Lesson Notes or on a student page. This one is referenced on page 8, problem 5.

Week #	Week beginning	Number of minutes spent doing cleanup	Week #	Week beginning	Number of minutes spent doing cleanup
1	August 17	20	10	October 19	10
2	August 24	40	11	October 26	20
3	August 31	30	12	November 2	60
4	September 7	0	13	November 9	50
5	September 14	45	14	November 16	40
6	September 21	55	15	November 23	30
7	September 28	50	16	November 30	40
8	October 5	20	17	December 7	20
9	October 12	30	18	December 14	0

Is Everette living up to his volunteering commitment?

Analysis will vary. Some possibilities: Notice that data is not really given by month, but rather by weeks for which months overlap. Estimation may be helpful. Some estimates of minutes per month:

- Weeks 1 – 3 (90 minutes) represent about $\frac{1}{2}$ of August.

  > Detailed answers support teacher instruction and student learning. Sometimes the answers provided surpass what may be expected from students.

  of a half-month's minutes. So, Everette is about 30 min

- Weeks 4 – 7 (150 minutes) represent almost all of Septe~~

  about 30 min over for September.

- Weeks 8 – 11 (80 minutes) represent almost all of October. This is $\frac{2}{3}$ (about 67%) of a month's minutes. Everette is about 40 min under for October.

- Weeks 12 – 15 (180 minutes) represent almost all of November. This is $1\frac{1}{2}$ (150%) of a month's minutes. Everette is about 60 min over for November.

- Weeks 16 – 18 (60 minutes) represent about $\frac{2}{3}$ of December. This is $\frac{1}{2}$ (50%) of a full month's minutes, and $\frac{2}{3}$ of 120 min is 80 min. Everette is about 20 min under for December.

Total over: 120 min   Total under: 60 min

Everette seems to be exceeding his requirement. On average, he is doing cleanup more than 120 minutes per month. In about $4\frac{1}{6}$ months he should do about 500 minutes, and he does 560 minutes. He averages about 134 minutes per month.
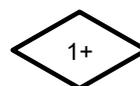
INTRODUCTION TO DATA ANALYSIS

We will find our "name scores" using the given values for each letter. Then we will create a human number line to help us organize our data. We will calculate measures of center and spread for the name score data, and understand how outliers may affect measures of center. We will identify characteristics of a statistical question.

[6.SP.1, 6.SP.2, 6.SP.3, 6.SP.5ac; SMP2, 3, 4, 5, 6]

GETTING STARTED

Dr. Dana is a veterinar~~...~~ records when they come to her clinic.

> Each lesson begins with this gray box containing a lesson summary and Common Core content and practice standards.
>
> Getting Started problems follow. They typically review or preview important content for the lesson.

1. What might Dr. Da~~...~~
    Possible answers inclu~~...~~

    What units of measure might she use?
    Inches or centimeters are appropriate for length, width, and height.
    Pounds or kilograms are appropriate for weight.

Here are the weights (in pounds) of Great Danes (a breed of dog) that came to her veterinary clinic in one week. Complete the table below using this data.

Great Danes (Males)		
45	125	140
190	150	

Great Danes (Females)		
130	125	90
100	120	95

	Males	Females
2. How many Great Danes came to the clinic?	5	6
3. Write the weights in order, from least to greatest.	45, 125, 140, 150, 190	90, 95, 100, 120, 125, 130
4. Record the lightest weight, in pounds.	45	90
5. Record the heaviest weight, in pounds.	190	130
6. What do you think is the typical weight? Explain your reasoning.	Answers will vary. One possible answer is 140 pounds because it is the middle of the ordered weights. The 45 lb dog was probably a puppy.	Answers will vary. One possible answer is 110 pounds because it is in the middle of the range.
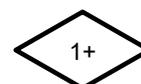
NAME SCORES

[SMP2, 3, 4, 5, 6]

Follow your teacher's directions. Answers will vary based on data collected.

(1) My first name: _____re: _____

> "Follow your teacher's directions" is a signal that some direct instruction is required. Look for the accompanying Slide Deck on the Teacher Portal, Lesson Notes on the following page(s), and an alternative page to the deck if some other lesson delivery method is desired.

(2) *Organize data.*
- Number of obser
- Make an organiz

(3) *Find measures of center.*

- Mode(s):

- Median:

> Notice the uncluttered, structured work space. Most *MathLinks* teachers do not require students to take traditional notes, since they are doing and discussing mathematics constantly, and all of the information students need is contained within the Student Packet.

(4) *Find values related to measures of sp*

- Minimum: _____   Maximum _____

- Range of a data set: _____ – _____ = _____
   max min

- Quartiles: Q_1 = _____ Q_2 = _____ Q_3 = _____

- Five-number summary: (_____ , _____ , _____ , _____ , _____)
   min Q_1 Q_2 Q_3 max
   (median)

- Interquartile range: _____ – _____ = _____
   Q_3 Q_1

(5) *Answer the question about a typical class name sc*
Answers will vary. Typically, the median is a good dat
score because it is a measure of center.

> A reminder that vocabulary should be recorded on page 0. New vocabulary is always underlined the first time it appears.

(6) Record the meanings of _____range_____, _____median_____, _____mode_____,

_____five-number summary_____, and _____interquartile range (IQR)_____ in **My Word Bank**.

> Here are some helpful icons. You'll need a few simple materials. Slide Deck 1.1a is available to use (or its alternative page), with corresponding Lesson Notes. Instruction is intended as "whole class." Three Reproducible pages Reproducible pages are available for making copies.

LESSON NOTES S1.1a: NAME SCORES

On slides, blue italic text suggests discussion; bl[...]

Students compute their "name scores" to gather [...] [...] [...]cabulary and concepts. Students create a human number l[...] [...]ith new terminology.

- Slide 1: **What is/are data?** Data are individual pieces of factual information recorded and used for the purpose of data analysis. It might be numerical (test scores, weights of dogs, name scores of students, number of days traveled each month) or categorical (colors, flavors of ice cream, music artists).

  Reveal the letter points.

  What do you notice about the chart? Students may see that commonly used letters have smaller values compared to less commonly used letters.

  What would be the name score for "JOEY"? Allow time for students to calculate his score.

  For (1), students calculate their own name score, then write their first name and the score on a sticky note.

We think the **bold italic** questions are good to ask. Modify them to suit your students' needs.

- Slide 2: Direct students to line up with their sticky notes in order based on their name scores. Discuss important statistics vocabulary and concepts using **R1-1 Five Number Summary Cards** and **R1-2 Measures of Center and Spread Cards**, and give the appropriate card to students as the discussion unfolds.

  - ✓ *How many students are in the lineup (the number of observations)?*
  - ✓ *Which name score represents the minimum value? The maximum value? What is the range of the data set?*
  - ✓ *Which name score(s) represent(s) the mode?*
  - ✓ *Which name score is the median?*
  - ✓ *What are quartiles? Which name score represents Quartile 1 (Q_1)? Q_2? Q_3? What's another name for Q_2?*
  - ✓ *What's the difference of Q_3 minus Q_1 (the IQR)?*
  - ✓ *Are any potential outliers in our data?*
  - ✓ *What is a typical name score for our class?*

Another Reproducible reminder.

LESSON NOTES S1.1a: NA
Continued

- Slide 3: Ask students to put their sticky notes on the wall or board, in order, and return to their seats. Then discuss the data set, statistics concepts, and vocabulary again.

  For (2), students record the number of observations (i.e., the number of sticky notes on the wall) and copy the name score data.

  Discuss measures of center. Review the meaning of mode (most common value(s) in the data set) and median (the middle value in an ordered data set). Note that if there are an odd number of observations, the median will be the one in the middle. If there are an even number of observations, it will be an average of the two middle data points. For (3), students record these values.

  Discuss measures of spread (variability). Calculate the range. Identify the data points in the five-number summary and the value of the interquartile range. For (4), students record these values. Use **R1-3 Blank Strips** for a visual explanation.

  If desired, informally discuss potential outliers (striking deviations from the center). Note that this will be addressed further in the next lesson.

  Finally, ask students to think about the statistical question at the top of the slide: "What is a typical name score for our class?" For (5), students answer this question, based on the data.

- Slide 4: For (6), introduce students to a vocabulary routine in the program. Periodically, students will be prompted to record explanations and examples of vocabulary in **My Word Bank** at the front of the unit. Give students time to do this, using knowledge from the activity and the Student Resources in the back of the unit to help them. Two examples are provided. **My Word Bank** should NOT be used to simply copy definitions from the back.

Problems that require student writing are numbered, and always in this blue color.

The same blue is used in italics for discussion questions.

SLIDE DECK ALTERNATIVE S1.1a: NAME SCORES

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

A = 1	B = 4	C = 4	D = 2	E = 1	F = 4	G = 3
H = 3	I = 1	J = 10	K = 5	L = 2	M = 4	N = 2
O = 1	P = 4	Q = 10	R = 1	S = 1	T = 1	
U = 2	V = 5	W = 4	X = 8	Y = 4	Z = 10	

Slide 1

(1)   Compute your first name score. Write your name and score on a paper or sticky note as directed.

Slide 2

Activity: Collect and discuss class data as directed by your teacher.

Slides 3

Explore this statistical question:
What is a typical name score for our class?

(2)   Organize your data.
- List the number of observations (n) in the population.
- List the observations, in order.

(3)   Find measures of center.
- mode(s)
- median

(4)   Find values related to measures of spread (variability)
- minimum, maximum, range
- five-number summary, interquartile range (IQR)

> As an alternative to using the slides, some teachers project this page on a white board or use a document camera.
>
> Note that all of the important parts of the slide decks are included on the alternative page.

(5)   Answer the statistical question above.

Slide 4

(6)   Record the meanings of <u>range</u>, <u>median</u>, <u>mode</u>, <u>five-number summary</u>, and <u>interquartile range (IQR)</u> in **My Word Bank**.

PRACTICE 1

Practice follows the introduction of a concept.

Bobbie likes to play card games with her friends. Listed below are the number of days she played cards each month last year. Let's explore: How often does Bobbie play cards?

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
5	10	8	7	7	6	5	5	8	0	8	10

Focus on organizing the data.

1. For how many months was data collected? (number of observations) n = __12__

2. Rewrite the data in order from least to greatest in the table below.

0	5	5	5	6	7	7	8	8	8	10	10

Focus on measures of center for the data.

3. Write the mode(s) of the data set: __5, 8__. What does this tell us about Bobbie's card playing habits? These are the most common number of days she played in a month.

4. The median of the data set is __7__. What does this tell us about her card playing habits? It is a measure of center. It is one way to describe the typical number of card-playing days in a month.

5. Which of these two measures of center best represents her card playing habits? Explain. Answers may vary. There are two modes so they are difficult to interpret. The median is between the two modes and also the middle data point, so it probably gives a better picture of card playing habits.

Focus on measures of spread for the data.

Watch for (all): Are students following along with all the new vocabulary?

6. Range: 10 – 0 = 10

7. Five-number summary (__0__, __5__, __7__, __8__, __10__)
    min Q_1 med. Q_3 max

8. Interquartile range (IQR): 8 – 5 = 3

9. What does interquartile range represent? Typically, half of the data fall in the IQR.

10. Would you say that Bobbie's card playing has a lot of variability? Justify your answer. Answers may vary. There is a fairly big range (10 days), but the IQR is only 3 days. Ignoring the extreme values, there is not much variability.

11. Use your data analysis to answer: How often does Bobbie play cards? Answers may vary. Possibilities are, she plays about 7 days per month, or she plays 5-8 days per month.

Problems 10 and 11 are suggested as possible journal ideas.

Appropriate for independent practice.

1+

Journal
Problems 10, 11

STATISTICAL QUESTIONS

Follow your teacher's directions for (1) – (3).

(1)   *What are two characteristics of a good statistical question?*
- The question is answered by collecting data.
- The data has potential for variability.

(2)   *Your teacher will give you some cards. Sort them and discuss. Then record here.*

These are … Statistical questions	These aren't… Statistical questions	I'm not sure
E I N O Q S T U (spells Q U E S T I O N)	A B C D F G H K L M P R	

(3)   *Choose two "these aren't" questions. Reframe them to be statistical questions.*
Questions will vary. Examples:
For P: What is the typical age of a president when inaugurated?
For D: What is the typical longevity of dogs that are the same breed as yours?

For each pair of questions, put a check next to the statistical question.

4.   What is the average length of hair for 6th graders in your school? ✓
How long is your hair?

5.   How far is it from Los Angeles to San Francisco?
How long does it typically take people to drive from Los Angeles to San Francisco? ✓

6.   How many points did Giannis Antetokounmpo score last night?
How many points per game can we expect Giannis Antetokounmpo to score? ✓

7.   How many blue whales can we expect to see on a whale-watching tour? ✓
How many blue whales did you see on your whale-watching tour?

8.   How tall are you?
Are you taller or shorter than the average height of students your age? ✓

9.   Do 6th graders today watch more TV compared to 6th graders 10 years ago? ✓
How much TV did you watch yesterday?

Let's revisit our name score activity again.

10.   What was a statistical question that we answered with the name score data?
Answers may vary. Example: What is the typical name score for students in our class?
We might try to infer: What is a typical name score for students in our school?

11.   Record the meaning of <u>statistical questions</u> in **My Word Bank**.

LESSON NOTES S1.1b: STATISTICAL QUESTIONS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Here we take a closer look at what makes a good statistical question.

- Slide 1: Reveal the description of a <u>statistical question</u> and discuss. Emphasize the academic vocabulary within the statement.

  What are two elements of a good statistical question?
  It is a question where information (data) is collected for the purpose of answering the question. It is a question where there is potential for variability (spread) in the data.

  What does variability mean? It means that different values could be in the data set. For example, a student might get different scores on tests, dogs may have different weights, someone may play cards different numbers of days each month, students may prefer different flavors of ice cream.

  For (1), students summarize the statement. Be sure they recognize that a statistical question has two components: it can be answered by collecting data AND data can take on different values.

  For (2), ask students to cut cards apart from **R1-4   These Are… These Aren't**, and sort them into piles based on their understanding of what a statistical question is. Discuss. The statistical questions spell the word "QUESTION."

  For (3), students rewrite two of the questions that are not statistical as statistical questions. Share and discuss.

Numbered problems not in parentheses (such as 4 – 11 in this activity) are not referenced in the slide deck, and are intended for practice individually or in groups.
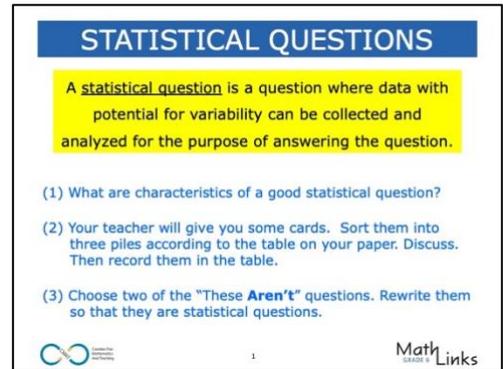
Examples of discussion questions to help students unpack vocabulary.

Example of a collaborative card sort that adds engagement and variety.

SLIDE DECK ALTERNATIVE 1.1b: STATISTICAL QUESTIONS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

> **A <u>statistical question</u> is a question where data with potential for variability can be collected and analyzed for the purpose of answering the question.**

(1)   What are characteristics of a good statistical question?

> Your teacher will give you some cards.
> Sort them into three piles according to the table on your paper.

(2)   Discuss how you sorted the cards. Then record them in the table.

(3)   Choose two of the "These Aren't" questions. Rewrite them so they are statistical questions.

MORE MEASURES OF CENTER AND SPREAD

We will revisit our class name score data to calculate the mean score and determine if our data set has potential outliers. We will find the mean absolute deviation (MAD) for data sets. We will then interpret measures of center and spread for data sets.

[6.SP.1, 6.SP.2, 6.SP.3, 6.SP.5abcd; SMP1, 2, 3, 4, 5, 6, 7]

GETTING STARTED

Answers may vary.

BK likes to talk about sports. She says that two attributes needed to be a good basketball team are the ability to play good defense and to get rebounds.

1. How might you measure the ability to play good defense?
   Look at the number of points the opposing teams score per game or on average.

2. How might you measure a team's ability to get rebounds?
   Look at the total number of rebounds they get per game or on average (offensive and defensive).

> Instructional idea:
> For problems that require students to explain their reasoning, consider doing some of these as pair-share activities. Discussing them first may improve the quality of written responses.

Two basketball teams' rebound totals for the last five games are

The Gremlins				
30	32	45	39	41

28	

3. Put a check next to the better statistical question. Explain.

> A "watch for" is a timely formative assessment heads-up where students may have misconceptions, make mistakes, or struggle with concepts.

...me?

...be *answered just by looking at one data point, and it could vary, depending on the interpretation of the data.*

> Watch for (4, 5): Do students put the numbers in order before trying to find the median?

4. Answer the statistical question from Problem 3. Explain your reasoning using measures of center and/or measures of spread.
   Gremlins' median number of rebounds is 39. Gladiators' median number of rebounds is 45. Using this measure, the Gladiators are rebounding better.

   The range of the Gremlins' rebounds is 45 – 30 = 15. The range of the Gladiators' rebounds is 61 – 21 = 40. This offers a lot of variability. The high number is quite a bit higher, and the low number is not much lower for the Gladiators, indicating that they may be rebounding better.

5. Sandy thinks that the median number of rebounds for both data sets is 45 because that is the middle number for both. Do you agree with Sandy? [SMP3]
   No. The median is the middle number when the data is IN ORDER from least to greatest. However, 45 happens to be the median number of rebounds for the Gladiators.
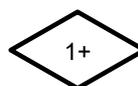
NAME SCORES REVISITED

[SMP2, 3, 4, 7]

Follow your teacher's directions for (1) – (7).

(1) *Describe how we estimated the mean (average) of our name scores during the activity.*
   We each started with the number of objects that equals each of our name scores. We first shared objects in our groups until each person had the same number of objects (or within one). We then repeated with other groups until everyone in the entire class had the same number of objects (or within one).

(2) *Find the mean for the class data set.*
   Answers will vary.

(3) *Find Waqueyzaquey's name score.*
   4 + 1 + 10 + 2 + 1 + 4 + 10 + 1 + 10 + 2 + 1 + 4 = 50

(4) *Match the phrase that best completes each prediction and discuss.* Probable answers:

   A → *IV*   B → *III*   C → *III*   D → *V*   E → *III*

Answers will vary.

	Median	**Mean**	**Mode**
(5) Measures of Center based on original Name Scores. *Copy from Name Scores.*			
(6) New Name Scores Measures of Center *Include Waqueyzaquey.*			

(7) *Explain which measure of center best reflects the class data set.*
   Answers may vary. Typically, the mean is more affected by a potential outlier, and Waqueyzaquey's name score likely represents an outlier. The median may be affected less or not at all. In these cases, the median usually best reflects a set of data. Decide what is true for your class.

8. Record the meanings of <u>mean</u> and <u>outlier</u> in **My Word Bank**.

LESSON NOTES S1.2: NAME SCORES REVISITED

Revisiting contexts is common when continuing concept development.

... uggests discussion; blue numbered text suggests written responses.

Students revisit the Name Scores activity to explore the class mean and potential outliers. If computational fluency interferes with the primary focus of instruction (the concept of mean), consider using a calculator. Division will be studied thoroughly in Unit 4.

- Slide 1: Focus attention back on student name score data.

  What single value do you think best represents the measure of center for the class? Answers will vary. Students may suggest the now familiar median or mode.

  Explain that we will now learn a third type of measure of center. Remind students that they computed their name scores in the previous lesson. Give each student the number of small objects (such as linking cubes or pennies) that are equal to the student's name score. Colors, sizes, etc. do not matter.
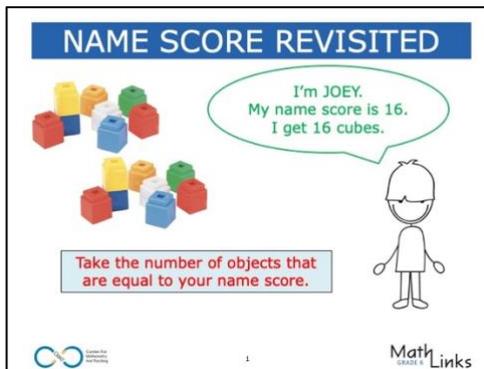
- Slide 2: Explain that students will share the objects among themselves so they all have the exact same amount. If this is impossible, no one can differ by more than one object. Demonstrate with the animated slide.

  If Ari shares with Al, how many cubes should he share so they have the same amount? 2. ***Now, if Tere shares with Al, how many should she give him?*** Probably 2.

  Continue animation until all stacks are the same (or within one). ***What does this tell you about the name scores of these students?*** When cubes are in equal stacks (or within 1), we have found an estimate for another measure of center called the <u>mean</u>. For these students, it is close to 7 cubes each.
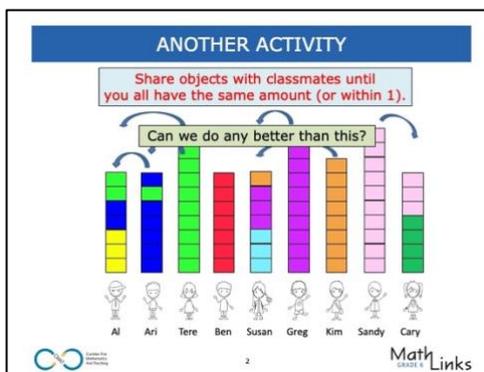
  Students now redistribute objects. One strategy is to have students share within their small groups first, and then small groups branch out to share with others, and so on until the whole class has shared. ***How many objects do you have now?*** Answers will vary.

  How does this number compare to the median and mode for our class set? Answers will vary. Most likely it is close to the median.

LESSON NOTES S1.2: NAME SCORES REVISITED
Continued

- Slide 3: Define <u>mean</u>. For (1), students describe how they found the estimated mean using objects. ***How would we compute mean if we didn't have objects?*** Give students time to conjecture and connect their knowledge of division to this concept. To find the mean, divide the total sum of all name scores by the total number of students. This is the average name score per student.

  For (2), students compute the mean. Calculators are recommended for large data sets. ***Is the computed value the same as the value we found with objects?*** When objects are distributed, everyone should have the same amount (the mean is this whole number) or be within 1 of one another (the mean is a mixed number or decimal). An example using arithmetic is provided on the slide. [SMP7]

- Slide 4: Introduce a new student to the class whose name is "Waqeyzaquey." For (3), students compute his name score.

  Distribute the **R1-5 Waqueyzaquey Card Sort** for a card sort. Ask students to work in groups to promote discussion. For (4), students predict how Waqueyzaquey's name score will affect measures of center. Discuss.

- Slide 5: Explain that a potential <u>outlier</u> in a data set is one that is very large or small compared to other values. Most likely Waqueyzaquey's name score fits this description. ***Are there other name scores in our class that might be outliers?*** Answers may vary.

  For (5), students record measures of center computed previously. For (6), they recompute with Waqueyzaquey included. ***Were your predictions correct?*** Discuss. Answers will vary.

  For (7), students consider which measure best reflects the new data set. Some may argue that the median is the best measure because it is resistant to outliers. Others may prefer the mean because it includes the new student.

SLIDE DECK ALTERNATIVE S1.2: Name Scores Revisited

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

A = 1	B = 4	C = 4	D = 2	E = 1	F = 4	G = 3
H = 3	I = 1	J = 10	K = 5	L = 2	M = 4	N = 2
O = 1	P = 4	Q = 10	R = 1	S = 1	T = 1	
U = 2	V = 5	W = 4	X = 8	Y = 4	Z = 10	

Slides 1-2

> Take the number of objects that is equal to your name score. Share objects with classmates until you all have the same number. If this is not possible, then share until all have within 1 of each other.

Slide 3

> **The <u>mean</u> is the arithmetic average of a data set.**

(1)   Describe how we estimated the mean (average) of our name scores during this activity.

(2)   Find the mean for the class data set using a computational procedure.

Slide 4

(3)   A new student named Waqueyzaquey enters our class. What is his name score?

(4)   Your teacher will give you some cards. Match the phrase that best completes each prediction. You may use a phrase more than once.

Slide 5

> **An <u>outlier</u> is a data value that is a "striking deviation" from the overall pattern of values in the data set.**

(5)   Copy the measures of center for the old class data.

(6)   With Waqueyzaquey's score in the class data set, determine the new measures of center.

(7)   Which measure of center best reflects the new data set? Explain.

PRACTICE 2

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions.   [SMP1, 2, 3, 4, 5, 6]

> See Program Information and the Teacher Portal (General Resources → Activity Routines) for more information and to prepare for using The *MathLinks* Rubric.

Diego, CA are
month. Here are

Aaron's number of Minutes				
35	35	35	35	0

John's number of minutes				
27	27	27	27	72

> Watch for (1): Do students correctly identify and calculate the three measures of center when they're all asked for on the same page?

1. For each student, find the mean, median, and mode. Label clearly.

	Mean	Median	Mode
Aaron	28	35	35
John	36	27	27

2. Do either of these data sets appear to include potential outliers? Explain.
   Aaron's 0 min and John's 72 min are both potential outliers, since they deviate quite a lot from the other numbers in their respective sets.

3. After 5 weeks, do these students appear to be meeting their volunteering commitments? Justify your decisions using the statistical measures you calculated and a written explanation. Answers may vary.
   If we assume 4 weeks per month (28 days, which is actually low for the typical month):
   Aaron volunteers 28 minutes per week, and 28 × 4 = 112. This is a little lower than being on pace, and will need to put in more hours than what he has been averaging.
   John volunteers 36 minutes per week, and 36 × 4 = 144. This is a little higher than being on pace, and does not need to put in more hours than what he has been averaging.

4. Aaron volunteered 0 minutes during week 5 because he was sick. What is the lowest number of minutes Aaron could volunteer week 6 to have an average of 30 minutes per week?
   Aaron would need 40 minutes week 6 to average 30 minutes per week.

   Say he reaches that number. Is this 6-week average on pace to reach the goal?
   If assuming 4 weeks per month, then Aaron is on pace to reach 120 minutes per month.

5. Go back to the opening problem, **Beach Cleanup**, and make changes or additions as needed, based on what you've learned about statistical measures.

> Going back to an Opening Problem is common after learning concepts and skills. Look for the heads-up in Lesson Notes or on a student page like this.
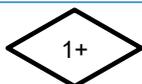
MEAN ABSOLUTE DEVIATION (MAD)

[SMP2, 3, 5, 6]

Recall the two basketball teams, the Gremlins and the Gladiators. Here are their scores over the last five games.

The Gremlins				
30	32	45	39	41

The Gladiators				
28	53	45	21	61

1. Graph the data points on the number lines below.

Gremlins Scores
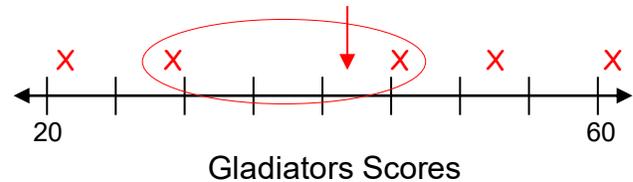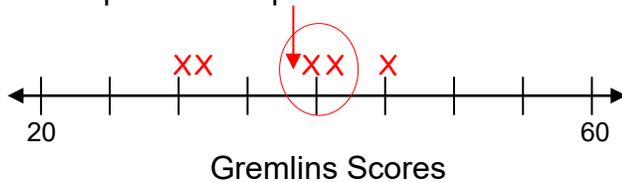
Gladiators Scores

2. Which scores seem to have greater spread (variability)? Justify your answer.

The Gladiators'. Their score range is 61 – 21 = 40 compared to the Gremlins' 45 – 30 = 15.

The <u>mean absolute deviation</u> (MAD) is another measure of spread for a data set. It is the arithmetic average of the distance of the data points to the mean.

Watch for (3): Do students follow the potentially complicated multi-step MAD process?

3. Compute the MAD statistic.

Calculating MAD	Gremlins	Gladiators
• Find the mean of the data. Mark it on the number line above with an arrow.	37.4	41.6
• Find the distance between each data point and the mean on the number lines.	7.4 5.4 7.6 1.6 3.6	13.6 11.4 3.4 20.6 19.4
• Find the sum of these distances to the mean.	25.6	68.4
• Find the mean (average) of these five distances. This is the MAD statistic. Circle data points that fall within this distance from the mean.	5.12	13.68

4. What do these MAD scores say about how spread out the scores are for these teams?

The Gremlins smaller MAD statistic means that the average distance of each data point from the mean is less, so the data clusters more about the mean.

Calculator   2+   R1-6   Journal Problem 4

PRACTICE 3

[SMP2, 3, 4, 5, 6]

This data shows the number of hours of online games played in one week by 13 teens.

13	28	15	10	10	17	4	15	17	10	8	11	6

1. Rewrite the data set in order from least to greatest in the table below.

4	6	8	10	10	10	11	13	15	15	17	17	28

2. Calculate measures of center and spread associated with the median.

> Consider having students collaborate while doing practice pages or problem-solving pages.
>
> Also consider pairs or trios working at vertical surfaces and transferring work to their Student Packets later.

        range __24__   median __11__

        five-number summary __(4, 9, 11, 16, 28)__

        interquartile range (IQR) __7__

3. What fraction of the observations falls in the IQR? One-half (50%).

4. Calculate measures of center and spread associated with the mean.

        mean __12.6__   mean absolute deviation (MAD) __4.5__

5. Does 28 appear to be an outlier? __yes__ How does this value affect measures of center?
   The outlier makes the mean have greater value than the median.

6. Use your analysis to answer: How many hours do teens typically spend playing online games?
   Answers will vary. Based on data collected and measures of center, we might infer that teens spend about 11-13 hours per week playing online videos. Based on measures of spread, about $\frac{1}{2}$ of teens typically spend 10-15 hours per week playing video games.

7. Record the meaning of <u>mean absolute deviation</u> in **My Word Bank**.
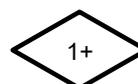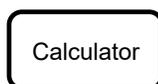
PRACTICE 4: EXTEND YOUR THINKING

[SMP2, 3, 5, 6]

This table shows heights of players on Basquiat High School's Football team.

height	5'5"	5'8"	5'9"	5'10"	5'11"	6'	6'1"	6'2"	6'3"
# of players	1	2	3	9	6	6	6	7	2

1. What characteristic (attribute) is described in the table? *height of players*

2. How is the characteristic measured? *feet and inches*

3. How many data points (observations) are in the table? *42*

> Extend Your Thinking pages may be good for the entire class. Or, when differentiation is needed, assign to some students for enrichment while others get review or intervention.

    What does this number represent? *the number of players on th...*

For the table below, use a calculator as needed. Explore the question "What is the typical height of a player on the football team?" with statistical measures. Hint: changing heights to inches may be easier.

4. Find measures of center and spread. Explain what they mean.

Statistical Measure	Value	Explanation
Range	10"	There is a difference of 10 inches between the tallest and shortest players on the team.
Median	5' 11$\frac{1}{2}$ " or 71.5"	When players stand in order from shortest to tallest, the middle player falls between 71" and 72".
Five-number summary	(in inches) (65, 70, 71.5, 73, 75)	These numbers relate to heights of players when they are lined up and broken into 4 equal groups by size.
Interquartile Range	3"	About half of the players are between 70" and 73" tall (the middle half).
Mean	71.5"	The arithmetic average (or average) height is 71.5 inches.
Mean Absolute Deviation	1.76"	It tells us there is large concentration of players who are betw... ...ch away from the...

> At the end of each lesson, this prompt reminds teachers to ask students to self-assess their progress on the lesson goals on the front of the Student Packet.

5. A new player joins the team and increases the ran... ...e the height of the player? *Either 5'3" (63 inches) or 6'5" (77 inches)...*

    What effect would this player have on the median a...
    Since the median is between 70" and 71", it would raise (or lower) it to the nearest whole inch. It would also move the mean slightly in the same direction.

DATA DISPLAYS

We will display data by constructing line plots (dot plots), histograms, and box plots (box-and-whisker plots). We will describe and analyze the overall shape of the data.

[6.SP.4, 6.SP.2, 6.SP.5abcd; SMP2, 4, 6]

GETTING STARTED

Here are three data displays for the same data set.

Line (or Dot) Plot

Histogram

Box (or Box-and-Whisker) Plot
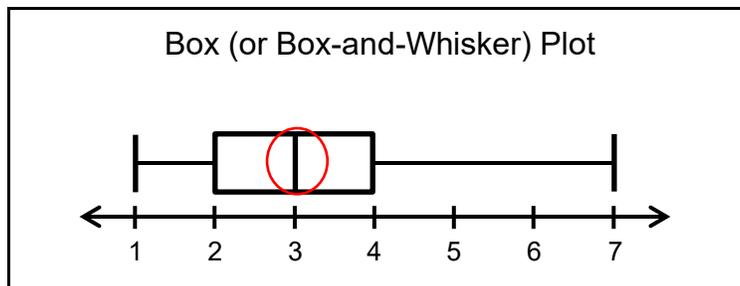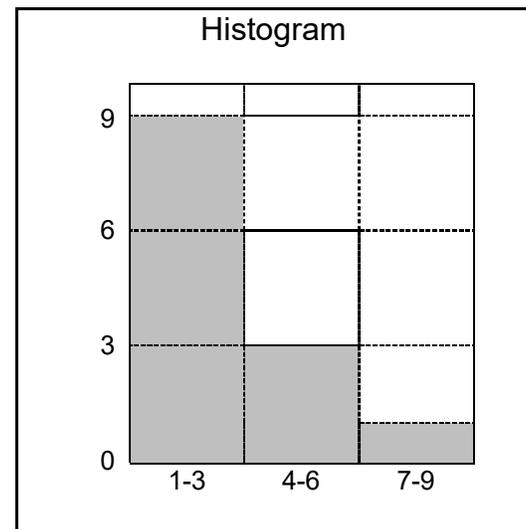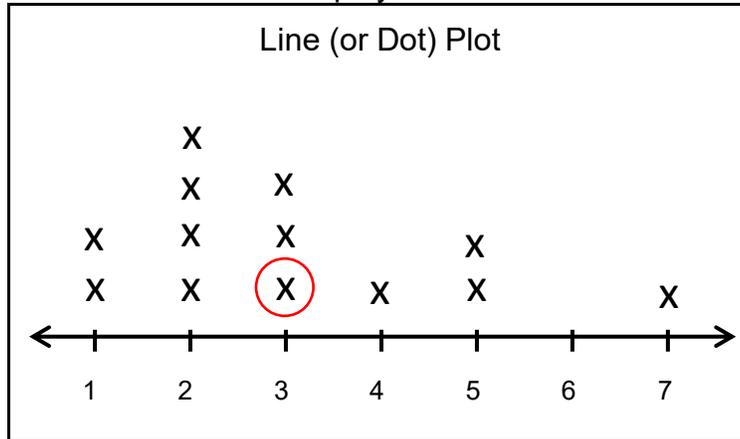
The mean and median are not represented on this display.

1. Make a list, in order, of the data values in the displays.
    1, 1, 2, 2, 2, 2, 3, 3, 3, 4, 5, 5, 7

2. Write the five-number summary for the data. Circle the median on each display if possible.
    (1, 2, 3, 4.5, 7)

3. Find the mean.
    $\frac{40}{13} = 3\frac{1}{13} \approx 3.07$

4. Which has greater value, the median or the mean? Why?
    The mean has greater value. The data has a little "tail" (the value of 7, visible mostly on the box and line plots), and the mean is typically pulled towards a tail.

THREE DATA DISPLAYS

Follow your teacher's directions for (1) – (5). *Displays will vary. Example intervals shown.*

(1) *Copy the class name scores data from Name Scores.*

(2) *Create a line plot of the name scores data. Number the tick marks. Title the display.*

0   5   10   15   20   25   30
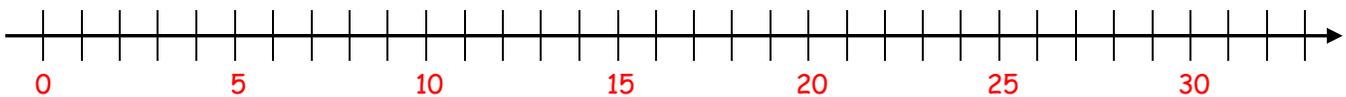
(3) *Create a histogram of the name scores data. Number appropriately. Title the display.*

Frequency

10
8
6
4
2
0

1-5   6-10   11-15   16-20   21-25   26-30
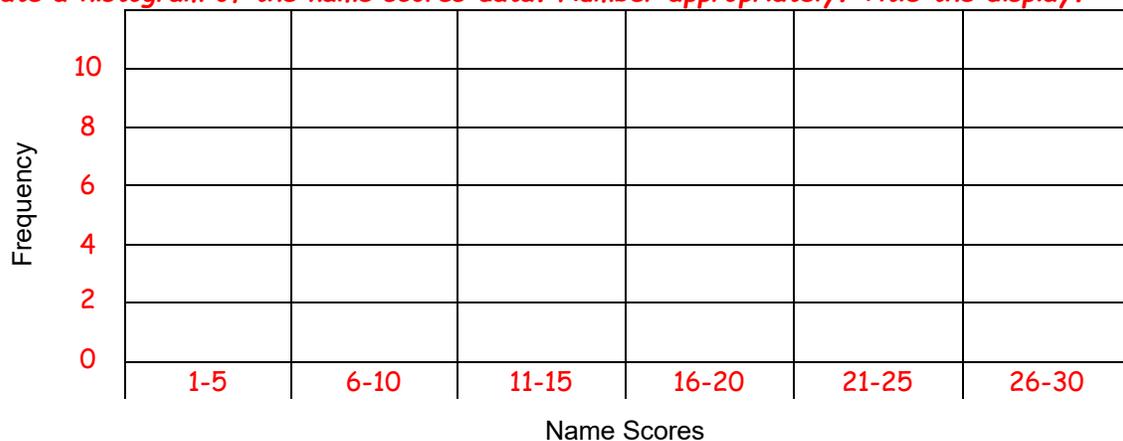
Name Scores

(4) *Copy the five-number summary for the name scores data from Name Scores.*

(_____ , _____ , _____ , _____ , _____)
   min Q_1 med Q_3 max

(5) *Use the five-number summary to construct a box plot. Number the tick marks. Title the display.*

0   5   10   15   20   25   30

6. Circle the words below that could be used to describe the shape of the data on our name score data displays. *Answers may vary.*

symmetric   gap   peak   cluster   tail   outlier

LESSON NOTES S1.3a: THREE DATA DISPLAYS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students use the class name scores data from **Name Scores** in Lesson 1 to create three data displays: a line (or dot) plot, histogram, and box (or box-and-whisker) plot. We also introduce vocabulary to describe data displays.

- Slide 1: Show the **Getting Started** line plot. *What are the features of a line plot?* It is a number line with X's stacked above the tick marks. The X's are lined up straight vertically and horizontally. We can also use dots if desired.

  Write the descriptive words "symmetric," "gap," "peak," "cluster," "tail," and "outlier" on the board. *How would you describe the shape of the data?* Possibilities include that there is peak at 2, a gap at 6, a cluster from 1-3, and a tail at 7. No outliers seem to exist.

  For (1) students copy the name scores data from **Name Scores**. *To make a line plot of our name score data, how should we label the number line?* Most likely, start at 0 and number by 1s or 2s. For (2), students create a line plot of the name score data. Help as needed.

  Which measure of center (mean, median or mode) is most easily seen on a line plot? The mode. *How would you describe the shape of the data using the words given previously?* Answers will depend upon the class data set.

- Slide 2: Show the **Getting Started** histogram. *What are the features of a histogram?* It looks like a bar graph organized into intervals of the same length instead of individual data points. There are no gaps between the bars.

  To make a histogram of our name score data, how should we label the bars? Answers will vary. Appropriate intervals may be by 3s, 4s, or 5s. Be sure students understand that all intervals must be the same size.

  For (3), students create a histogram of the name score data. Help as needed. Then, ask students who used different intervals to share their displays. *How does choice of interval affect the look of the display?* Answers will vary.

  Are there any clusters, gaps or peaks within the histogram? Is there symmetry? Answers may vary.

LESSON NOTES S1.3a: THREE DATA DISPLAYS
Continued

- Slide 3: Show the **Getting Started** box plot.

  What do you notice about a box plot? It is on a number line, but has a rectangle with lines attached to it. It is appropriate to identify the "box" and "whiskers" at this time.

  Which of the words from before apply to this display? It's the same data, and we can still see the tail toward 7, and a cluster from 1-3. Recall that about 25% of the data points lie in each "whisker" and each part of the "box," so about 50% of the data should be from 1-3 (it's actually more).

  For (4), students copy the five-number summary from **Name Scores**. For (5), students create a box plot of the name score data. Help as needed.

  Based on our five-number summary, what should our scale be for the number line? Answers may vary, but students will probably use the same scale as the line plot.

- Slide 4: Discuss the meaning of the descriptive words if they have not already been introduced. Then reveal line plots one at a time and discuss.

  Describe Plot A. It appears to be symmetrical, with small gaps, but no tails, or outliers.

  Describe Plot B. It appears to cluster to the right and have a tail to the left.

  Describe Plot C. It has a gap which may indicate an outlier. It clusters at the lower values.

  Describe Plot D. This is symmetrical with a peak near the middle.

SLIDE DECK ALTERNATIVE S1.3a: THREE DATA DISPLAYS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

(1) Copy our class name score data from **Name Scores**.

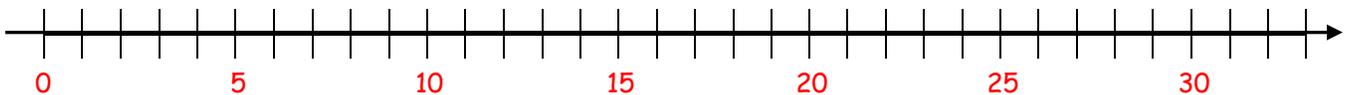(2) Create a dot (or line) plot with our name score data. Number the tick marks. Title the display.

Slide 2

(3) Create a histogram using our name score data. Number appropriately. Title the display.
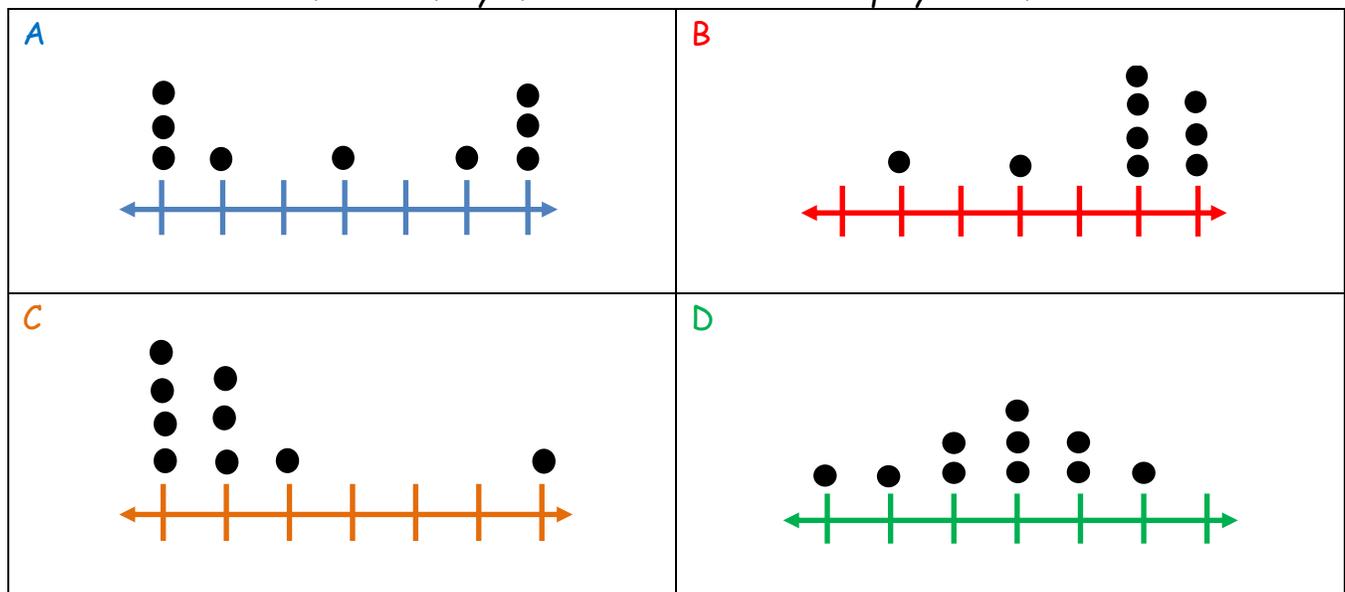
What intervals make sense for this data?

Slide 3

(4) Copy the five-number summary from our name score data.

(5) Create a box plot using this five-number summary. Number the tick marks. Title the display.

Slide 4

Use words such as symmetry, gap, peak, cluster, tail, and outlier.
How would you describe each data display below?

PRACTICE 5

Recall the number of hours of online games reported by teens in one week.

13	28	15	10	10	17	4	15	17	10	8	11	6

1. Record the data, in order, from least to greatest, and write the five-number summary.

4, 6, 8, 10, 10, 10, 11, 13, 15, 15, 17, 17, 28
Five-number summary: (4, 9, 11, 16, 28)

> Watch for (all): Do students draw equally spaced tick marks on number lines? For the line plot, if x's are not drawn at the same size or not lined up horizontally, it may be easy to confuse which numbers.

2. Construct a line plot of the gaming hours and label appropriately.

Friends Online Gaming Hours

3. Construct a histogram of the gaming hours and label appropriately.

Friends Online Gaming Hours

4. Construct a box plot of the gaming hours data.
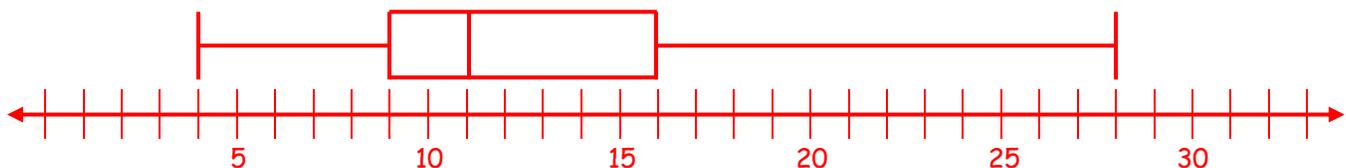
5. Describe the shape of the data. Use words such as symmetry, gap, peak, cluster, tail, or outlier if appropriate.

The spread of online gaming data is from 4 to 28 hours and clusters around 10-15 hours. There is a gap from 18 to 27 hours, and 28 is potential outlier.

PRACTICE 6: THE FOOTBALL TEAM REVISITED

[SMP2, 4, 6]

Recall the heights of players on Basquiat High School's Football team.

height	5'5"	5'8"	5'9"	5'10"	5'11"	6'	6'1"	6'2"	6'3"
# of players	1	2	3	9	6	6	6	7	2

1. Make a line (dot) plot of heights of the team. Describe the shape of the data, based on this plot.

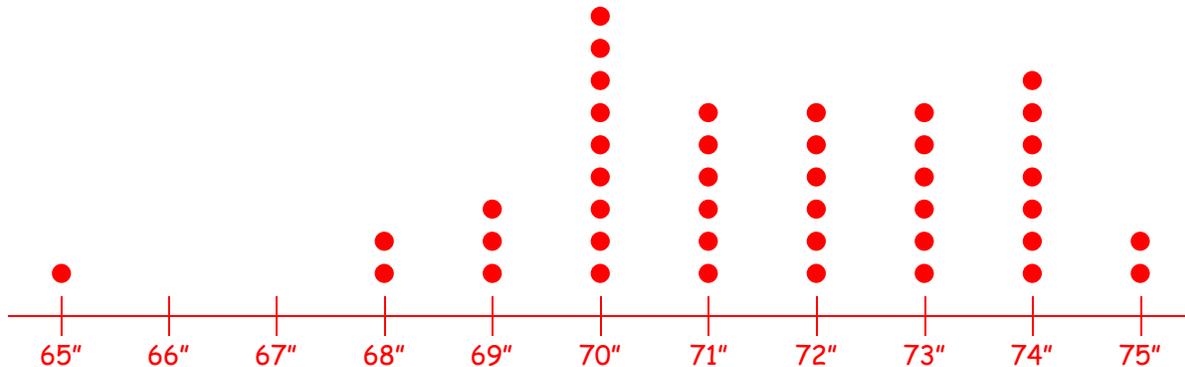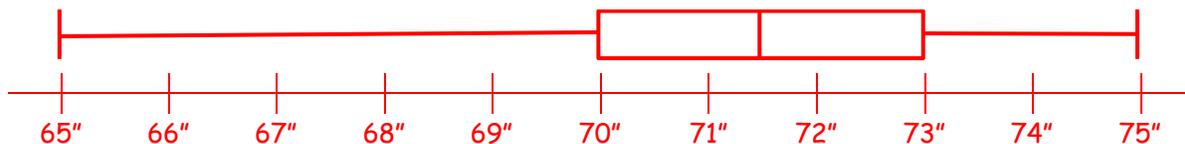It is fairly symmetric. Most of the data clusters from 70 inches to 74 inches. It tails a little to the left.

2. Make a box plot of heights of the team. Describe the shape of the data, based on this plot.
   Five-number summary is (65, 70, 71.5, 73, 75).

The range of the data is 10. About 50% of the data fall between 70" and 73".

3. Which statistical display do you think best helps you to describe the shape of the data? Why?
   Answers will vary. Dot plots are cumbersome with large data sets.
   The box plot provides a clearer picture of the center and spread.

4. Answer this statistical question by referring to displays that you think are most useful: What is the typical height of a player on the Basquiat High School football team? Why?
   Answers will vary. Based on the box plot, the median height of a player on the football team is 71.5".
   The mean height is also 71.5". The mode is 70", which is very close to these other measures of center. About half of the team has a height from 70" to 73".
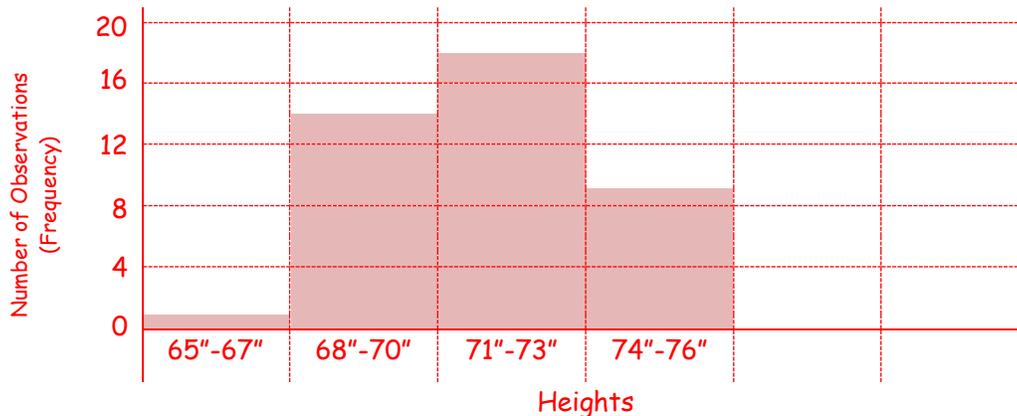
1+

INTERPRETING HISTOGRAMS

[SMP2, 4, 6]

Follow your teacher's directions for (1) – (2).

(1) *Create a histogram of the team's height data from the previous page.*
     Use intervals of 2, 3 or 4.
     Answers will vary, depending on where the intervals start and size of interval. One option is shown.

(2) *Describe the shape of the data.*
     Answers will vary, depending on where the intervals start and size of interval. For the histogram
     above, the data is most concentrated between 68" and 73" with a small peak from 71" to 73".

Chris is training for a half-marathon in the fall. The histogram below shows the number of
minutes Chris ran each training day over the summer.

3. Can you tell from this display how many
    minutes Chris ran each day? If so, how
    many? If not, why not?
    No. Histograms give intervals, not exact minutes,
    so we cannot tell from the display precisely how
    many minutes he ran each day.
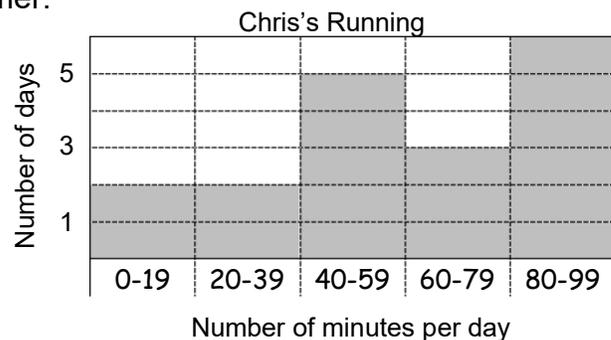
4. Can you tell from this display how many days Chris ran?
    If so, how many? If not, why not?
    Yes. The vertical column gives frequencies. He trained 18 days.

5. Based on this display, describe Chris's typical training schedule.
    From the histogram, most often he either ran 40-59 minutes or 80-99 minutes.

6. Why is it important to pay attention to histogram intervals?
    Even with the same data set, different intervals can lead to different shapes of the histogram. This
    may give different impressions of the meaning of the data.

LESSON NOTES S1.3b: INTERPRETING HISTOGRAMS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students create and interpret histograms. They explore how intervals can affect the "story" they tell.

- Slide 1: Students refer to the football team height data from **Practice 6**.

    Where will height go on the histogram? The horizontal axis. ***What could be appropriate intervals?*** Probably intervals of 2, 3, or 4 inches. Be sure students understand that intervals must start at some number and then be the same width.

    Where do we record the number of observations for each interval (or frequency)? On the vertical axis. ***What would be appropriate intervals?*** Skip counting by 3, 4, or 5 will work, depending on frequencies.

    For (1), students choose appropriate inte... histogram. Encourage them to choose inc... the histogram comparison discussion tha... they describe the shape of their data.

    Ask students to compare histograms wit... may observe that the choice of interval... the data display tells.

> The students in our "*MathLinks* class" appear on the front of every Student Packet in grades 6 – 8. They portray a wide range of cultures and qualities. These students often share their thinking, their work, or their errors on slides, which is intended to stimulate class discussions. They typically do not appear in Student Packets to minimize clutter and maximize work space on student pages.

- Slide 2: **Critique Student Work** is a recurring feature of the program intended to stimulate discussion. Reveal histograms drawn by Emmett and Zara.

    What is the same about the height intervals in these histograms? They both displayed height in intervals of 5.

    What is different about the height intervals? One starts at 61", and the other starts at 65".

    What do you notice about the shape of the data? By just altering the starting point of the intervals, the graphs appear to tell a different story. Emmett's graph suggests that most of the athletes are 71" or taller. Zara suggests that most of the athletes are from 70" to 74". While these are not inconsistent statements, they emphasize different ways to interpret the data.

SLIDE DECK ALTERNATIVE S1.3b: INTERPRETING HISTOGRAMS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.
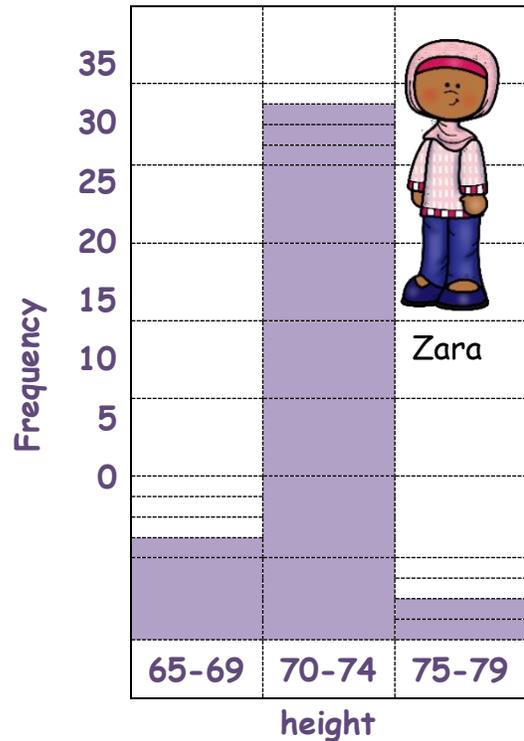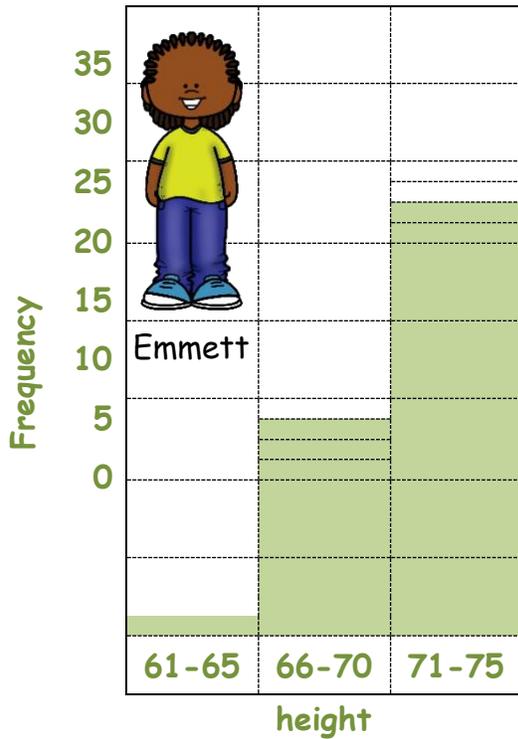
Slide 1

(1) Create a histogram of the football team height data using intervals of 2, 3 or 4.

(2) Describe the shape of the data

Compare your histograms with classmates.
How are they the same? How are they different?

Slide 2

Emmett and Zara drew height data histograms. Critique their work.

What do these histograms suggest about the height data?

Why are the stories different?

INTERPRETING BOX PLOTS

Follow your teacher's directions for (1) – (5).

(1) *Write a five-number summary for the box plot.*

(__0__ , __15__ , __23__ , __25__ , __27__)

0   5   10   15   20   25   30
Number of Texts

(2) *Create a data set that COULD match this box plot. Write values in order from least to greatest.*
Answers may vary greatly. Data points at quartiles (underlined) must correspond to the five-number summary. Example:

0, 0, 0, 1, 2, 14, 16, 17, 18, 19, 20, 20, 23, 24, 24, 24, 24, 25, 25, 25, 26, 27, 27, 27, 27

(3) *Explain who is correct. Use the data and display to justify your reasoning.*
Anita is correct, although she did not explain why Gerry is mistaken. The box plot divides the data set into 4 equal parts. About one-fourth of the data falls in the light rectangle, and one-fourth falls in the dark rectangle. Therefore, about the same number of 6th graders should be represented in each interval. A bigger box simply means that the data is more spread out.

A *MathLinks* class error analysis opportunity.

(4) *Make a histogram of YOUR data set.*
Intervals and displays will vary. Example based on sample data above:

6th Graders Texting Behavior

Frequency

6

4

2

0-4   5-9   10-14   15-19   20-24   25-29

Number of texts

(5) *Interpret the meaning of the data through the shape of the histogram. How does this compare to your interpretation of the box plot?*
Answers may vary somewhat depending on the data set created. The histogram above clumps at the right, showing that most 6th graders texted between 20-29 times. However, the box plot shows that half of those surveyed texted between 15-25 times. These are not inconsistent statements, but they do emphasize different parts of the data set in the interpretation.

S1.3c   2+

LESSON NOTES S1.3c: INTERPRETING BOX PLOTS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students interpret and draw conclusions from box plots. They continue to observe that the "story" told by data sometimes depends on how it is presented and displayed.

- Slide 1: Show introductory information and the box plot. For (1), students create a five-number summary of the plot.

  How many students participated in the survey? 25

  Do we know how many texts each student sent? No

  What do we know about the median? It's the middle value of an ordered list. It appears to be 23.

  What do we know about Q_1? It falls between two observations (when sequenced in order). Two consecutive values are both 15 or they average 15.

  For (2), ask students to create a data set that could match the plot. Share. Discuss similarities and differences.

- Slide 2: Again, **Critique Student Work** is included to stimulate discussion. Pose Gerry's and Anita's statements. Possible discussion starters:

  What do we know about the IQR? About half of the data fall in it.

  What fraction of the data fall in each part of the box of the IQR? About one-fourth

  For (3), students use ideas generated in the class discussion to explain who is correct, in writing.

- Slide 3: For (4) and (5), students create and interpret a histogram and compare histograms and the box plot.

  Do all of your histograms tell the same story? Most likely they vary some, based on data sets created and size of intervals.

  Do they tell the same story as the box plot? Possibly, or they may emphasize different aspects of the data set.

The slide that contains the error analysis prompt.

SLIDE DECK ALTERNATIVE S1.3c: INTERPRETING BOX PLOTS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

Twenty-five 6th graders reported the number of texts
they sent one evening. Here is a box plot of the results.

(1)   Write a five-number summary for the box plot.

(2)   Create a data set that COULD match this box plot. Write values in order from least to greatest.

Compare data sets with classmates.
How are they the same? How are they different?

Slide 2

Gerry thinks that more students texted 15-23 times than 23-25 because the box to the left is bigger than the box to the right. Alexandra disagrees.

(3)   Explain who is correct. Use the data and display to justify your reasoning.

Anita   Gerry

Slide 3

(4)   Make a histogram of your data set.

Compare your histograms with classmates.
How are they the same? How are they different?

(5)   Interpret the meaning of the data set through the shape of the histogram. How does this compare to your interpretation of the box plot?

PRACTICE 7

Let the data displays on **Getting Started** represent data collected about the number of hours teens typically play sports on the weekend.

1. How many teens were surveyed? Explain how you know.
   We can tell from the line plot and histogram that she surveyed 13 people.

2. What is the typical number of hours played? Explain why you think this.
   Using the median from the box plot, the typical number of hours might be 3.
   From the line plot and box plot, we see that most teens played between 2 and 5 hours.

3. Do you think that anything about these displays is misleading? Explain.
   Answers may vary. The histogram and line plots indicate that most teens played sports between 1-3 hours, while the box plot indicates that about 50% of teens do this.

The box plot below contains only whole number values. Use the box plot for problems 4 – 9.
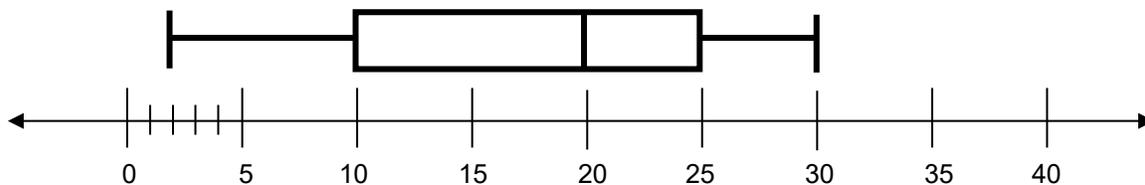
4. Create an appropriate title for this box plot if the data represents prices.
   Answers will vary. One possible answer: Prices spent on presents for friends over the year.

5. What is the range of the data set? The range is 28.

6. List the five-number summary. (<u>2</u> , <u>10</u> , <u>20</u> , <u>25</u> , <u>30</u>)
           min Q_1 med. Q_3 max

7. The median does not split the box into two identical rectangles. Explain what this means.
   About one-fourth of the data fall in each rectangle. The number of observations in the second quartile (left rectangle) are more spread out than the number of observations in the third quartile (right rectangle).

8. Why can't we determine the mean from the box plot? We don't have the exact data points to calculate the mean. However, we can assume the mean is less than 20 since the data points are more spread out to the left (a tail is to the left).

9. What conclusions can you draw from the shape of the graph? Answers will vary. Examples: Prices range from $2 - $30. One-fourth of the prices are in each range: $2-$10, $10-$20, $20-$25, and $25-$30. The data appears to tail to the lower prices and probably clusters more towards the higher prices.

REVIEW

POSTER PROBLEMS: STATISTICS

See Activity Routines on the Teacher Portal for directions.   [SMP2, 4]

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.

- Each group will start at a numbered poster. Our group start poste

- Each group will have a different colored marker. Our group mark

> Poster Problems is an Activity Routine that appears in the Review section in every unit in grades 6 – 8. It is an engaging way for students to solve problems and analyze work together. When teachers watch and listen, they are able to gauge student understanding and identify areas for review or reteaching. By establishing classroom norms, all students will find a safe place to contribute ideas and collaborate with peers.

Part 2: Do the problems on the posters by following your teacher's di

Poster 1 (or 5)	*Shop Shoes* sold the following sizes during t 9, 7, 8, 8, 10, 8, 6, 5, 9
Poster 2 (or 6)	Below are the housing prices (in thousands) sales in Mathville. $475, $470, $460, $375, $500, $450, $6
Poster 3 (or 7)	Teens were surveyed on the number of hou looking at a screen. 63, 50, 40, 15, 35, 45, 54, 29, 2
Poster 4 (or 8)	The number of pets students own are show 3, 4, 2, 0, 1, 2, 12, 4, 2, 3, 5,

Possible answers on the next page.

A. Copy the data in numerical order and determine the median and mode.

B. Determine the mean for the data set and note any potential outliers.

C. Find the five-number summary for the data set.

D. Make a data display that would be appropriate for the data set. Be sure to label the graph.

Part 3: Return to your seats with your original poster. Work with your group.

Write a statistical question that can be answered with your group's data set and display. Answer the question and explain your thinking using the measures of center, variability, and/or the data display.

POSTER PROBLEMS: STATISTICS
Answer Key

Poster 1 (or 5)	Poster 2 (or 6)
A. 5, 6, 7, 8, 8, 8, 8, 9, 9, 10 Median = 8 Mode = 8	**A.** (all values in dollars) 375, 410, 450, 460, 470, 475, 480, 500, 500, 650 Median = 472.5 Mode = 500
B. Mean = 7.8	**B.** Mean = 477
C. (5, 7, 8, 9, 10)	**C.** (375, 450, 472.5, 500, 650) 650 is a potential outlier.
D. Displays will vary. Line (or dot) plot, box plot, or histograms are expected.	**D.** Displays will vary. Line (or dot) plot, box plot, or histograms are expected.
Poster 3 (or 7)	**Poster 4 (or 8)**
A. 15, 25, 29, 35, 37, 38, 40, 45, 49, 50, 54, 63 Median = 39 Mode - none	**A.** 0, 0, 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 4, 5, 12 Median = 2 Mode = 2
B. Mean = 40	**B.** Mean = 3 12 is a potential outlier
C. (15, 32, 39, 49.5, 63)	**C.** (0, 1, 2, 4, 12)
D. Displays will vary. Line (or dot) plot, box plot, or histograms are expected.	**D.** Displays will vary. Line (or dot) plot, box plot, or histograms are expected.

MATCH AND COMPARE SORT: STATISTICS

See Activity Routines on the Teacher Portal for directions.   [SMP6]

1. Individually, match words with descriptions. Record results.

Card set △			Card set ◯		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I	mean	B	**I**	median	C
II	mean absolute deviation	D	**II**	interquartile range	D
III	dot plot	C	**III**	box plot	A
IV	measures of center	A	**IV**	measures of spread	B

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different. *Choice of vocabulary words to comp* [...]
   One possible answer:

mean

Match and Compare Sort is an Activity Routine that appears mostly in the Review section in several units in grades 6 – 8. As a card sort, it's an engaging way for students to collaborate with peers while reviewing important concepts and vocabulary from the unit in a safe environment.

Outliers can influence it

Requires addition and division to compute

Measure of center of a data set

Resista[...]

Identify by listing data values in order and then finding the middle value

3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

WHY DOESN'T IT BELONG?: STATISTICS

See Activity Routines on the Teacher Portal for directions.
Use the data set with eight data points below, and the box with
four numbers to the right.

2 4 6 8 10 12 14 18

All responses below should include statistical concepts and
vocabulary.

A	B
9.25	39
C	**D**
12	5

1. Choose each of the four numbers in the box and explain why it does not belong with the
    other three in relation to the given data set. *Answers will vary. Some possible answers:*

A	B
9.25 is the only one of these numbers that represents a measure of center of the data set (mean).	*39 is the only one of these numbers that is outside the range of the data set. If considered along with the other numbers in the data set, we might say it's a potential outlier.*
C	D
12 is the only one of these numbers that is an actual data point in the data set.	*5 is the only one of these numbers that represents a quartile of the data set (Q$_1$).*

Why Doesn't it Belong? is an Activity Routine that appears mostly in the Review section in several units in grades 6 – 8. It promotes creative and divergent thinking, and encourages mathematical reasoning and justification.

2. Make any data display using ⬚⬚⬚⬚⬚⬚⬚⬚ at the top of the page. Create a
    reasonable context for these ⬚⬚⬚⬚⬚⬚⬚⬚ display appropriately.
    Answers will vary. As an example, a box plot could be constructed for the ages of eight cousins in a family.

VOCABULARY REVIEW

The crossword grid contains the following answers:

Across:
1. BOX
6. MAD
7. STATISTICAL
8. DOT
9. MEDIAN
11. RANGE

Down:
2. OUTLIER
3. MODE
4. QUARTILES
5. FIVE
6. MEAN
10. IQR
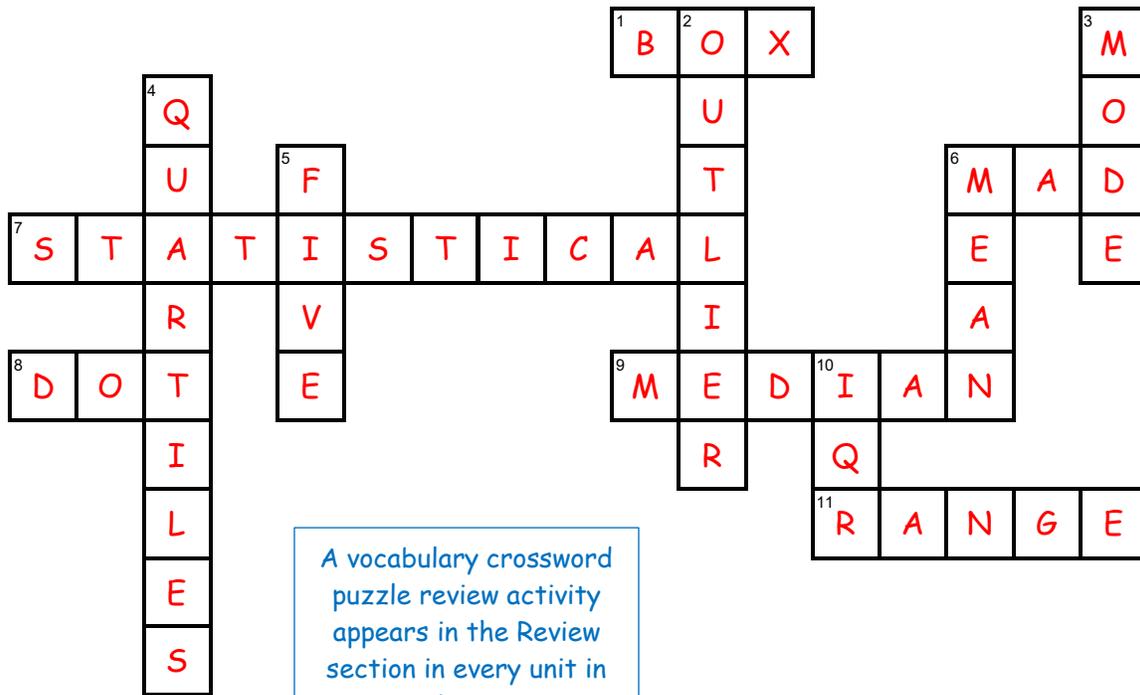
A vocabulary crossword puzzle review activity appears in the Review section in every unit in grades 6 – 8.

Across

1   plot based on the five-number summary

6   average of the distances of data points from the mean (abbreviation)

7   a question where data that has potenti al for variability, and can be collected and analyzed

8   plot that shows data frequencies above a number line

9   middle number of an ordered data set of data

11 difference between greatest and least value in a data set

Down

2   value that has striking deviation from others in a data set

3   most common value in a data set

4   data points that divide an ordered data set into four equally-sized groups

5   number of values in a data set summary that includes quartiles

6   the arithmetic average

10 difference between Q_3 and Q_1 (abbr.)
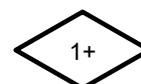
SPIRAL REVIEW

1. **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

    a. Begin with the number 2. Multiply your number by 6. Multiply the result by 7. Multiply that result by 8. Multiply that result by 9. (You should have a "big number.")
    I began with the number ____2____. After multiplying, my big number is ____6,048____.

    b. Start with your <u>big number</u> from above. Divide it by 6. Divide that result by 7. Divide that result by 8. Divide that result by 9. After dividing, I got ____2____.

    (Organize and show work below.)

> This Activity Routine appears in Spiral Review for every 6th grade Student Packet to help address grade level computational fluency goals. Students practice the four operations on whole numbers and decimals throughout the course.
>
> One or more Spiral Review pages appear in every Student Packet. These have been carefully designed to distribute practice so that important mathematical ideas remain fresh for students. See Program Information charts for more details about when topics are reviewed. Note that this is one place where we list *topics* that are reviewed, *not standards*.

SPIRAL REVIEW
Continued

2. Sage and Mason were playing a video game. Sage started with 1,345 points. Sage then lost 329 points, gained 415 points, lost a third of his points and then doubled his points. Mason started with 580, gained 1,002 points, lost half the points, and gained 272 points. Who had the most points at the end?

   Sage – 1,908, Mason – 1,063;
   Sage had the most points.

3. Sookie went to the craft store to buy items for her project. She bought 2 containers for $2.75 each, three-fourths yard of fabric at $7.00/yard, one box of buttons for $3.42 and a dozen foam balls for $0.78 each.

   a. Write a numerical expression for the total bill.

      $2(2.75) + \frac{3}{4}(7) + 3.42 + 12(0.78)$

   b. Simplify the expression.

      $23.53

SPIRAL REVIEW
Continued

4. Kristi is painting a board for a home improvement project, shown on the right.

   $2\frac{1}{2}$ ft

   $8\frac{1}{5}$ ft

   a. What is the area of the board?

      $20\frac{1}{2}$ square feet

   b. What is the perimeter of the board?

      $21\frac{2}{5}$ feet

   c. What is the difference between the larger side and the smaller side?

      $5\frac{7}{10}$ feet

5. Write 3 equivalent fractions following the directions below.

	Write the fraction in its simplest form.	Write an equivalent fraction with 24 as the denominator.	Write an equivalent fraction with 18 as the numerator.
$\dfrac{3}{4}$	$\dfrac{3}{4}$	$\dfrac{18}{24}$	$\dfrac{18}{24}$
$\dfrac{2}{12}$	$\dfrac{1}{6}$	$\dfrac{4}{24}$	$\dfrac{18}{108}$
$\dfrac{3}{6}$	$\dfrac{1}{2}$	$\dfrac{12}{24}$	$\dfrac{18}{36}$
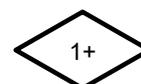
REFLECTION

Identifying Big Ideas and their connections helps students view mathematics as a cohesive and connected body of knowledge. It also makes learning more meaningful, and efficient.

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in connections that you noticed.

Investigate concepts and solve problems involving length, area, and volume. (6.G.A)

Extend the number system to include negatives. (6.NS.C)

Use statistical measures and displays to describe center and spread. (6.SP.AB)

Explore relationships between inputs and outputs. (6.EE.C)

Gain computational fluency with positive rational numbers. (6.NS.AB)

Rewrite and evaluate expressions and solve equations. (6.EE.AB)

Explore and apply ratio and rate reasoning and representations. (6.RP.A)

Give an example from this unit of one of the connections above.
- Use algebraic representation s to create geometry formulas for area and volume
- Use knowledge of volume plus computations to determine costs for the food bank

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Why do you think we revisited data sets in this unit [SMP 4]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.
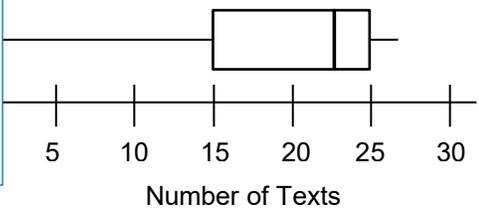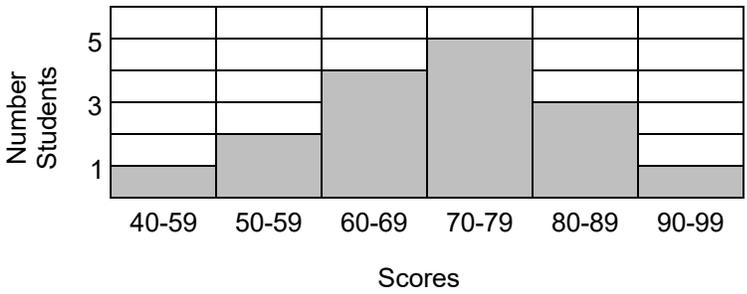- It is easier to do more math with a concept once it is understood
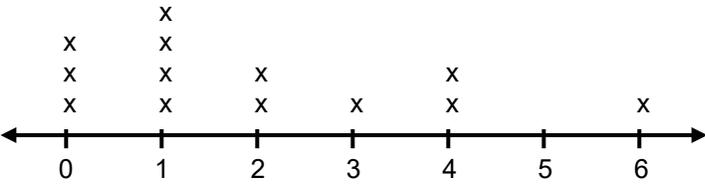- Opportunities are given to see data in different ways

4. **Making Connections.** Explain how you communicated a story with data.
- Used statistical computations and displays to analyze data for topics such as beach cleanup
- Learned that data can be misleading, depending on how it is presented

> This section is available on the public website for parents and others who support students.

STUDENT RESOURCES

Word or Phrase	Definition
box plot	A box plot, or box-and-whiskers plot, is a graphical representation of the five-number summary of a data set. See five-number summary. Plot of Number of Texts Per Day of 6th Graders Number of Texts
dot plot	A dot plot is a graphical representation of a data set where the data values are represented by dots above a number line. See line plot.
five-number summary	The five-number summary of a data set consists of its minimum value (min), first quartile Q_1, median Q_2, third quartile Q_3, and maximum value (max). The five-number summary is usually written in the form (min, Q_1, med., Q_3, max). The five-number summary of the data set 1, 1, 1, 3, 5, 5, 6, 7, 23 is given by (min, Q_1, med., Q_3, max) = (1, 1, 5, 6.5, 23).
histogram	A histogram is a graphical representation of frequencies of a numerical variable using rectangles. For a histogram, the horizontal axis is divided into intervals. Each interval forms the base of a rectangle whose height corresponds to the frequency of values of the variable in that interval. Quiz Scores of a Class of 16 Students Number Students / Scores: 40-59, 50-59, 60-69, 70-79, 80-89, 90-99
interquartile range	The interquartile range (IQR) of a numerical data set is the difference between the third quartile and the first quartile of the data set. The interquartile range is a measure of the variation of the data set. For the data set 1, 1, 1, 3, 5, 5, 6, 7, 23, Q_1 = 1, Q_3 = 6.5, and IQR = 5.5

The box shown includes precise definitions. When they first arise in a unit, teachers help their students unpack these definitions and record them in My Word Bank using their own words, examples, and/or diagrams.

Word or Phrase	Definition
line plot	A <u>line plot</u> is a graphical representation of a data set where the data values are represented by marks, such as dots or X's, above a number line. See <u>dot plot</u>. Line Plot of Number of Pets for 13 Students $$\begin{array}{cccccccc} & & x & & & & & \\ & x & x & & & & & \\ x & x & x & x & & x & & \\ x & x & x & x & x & x & & x \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & & 6 \end{array}$$ Number of Pets
mean	The <u>mean</u> of a data set is a measure of center equal to the average of the values in the data set. The mean is calculated by adding the values in the data set and dividing by the number of data values. The mean of the data set 1, 1, 1, 3, 5, 5, 6, 7, 23 is $$\frac{1 + 1 + 1 + 3 + 5 + 5 + 6 + 7 + 23}{9} = 5\frac{7}{9} = 5.77...$$
mean absolute deviation	The <u>mean absolute deviation</u> (MAD) of a data set is the average of the (positive) differences between the values in the data set from the mean. The MAD is a measure of the variation of the data set. For the data set {3, 3, 5, 6, 6}, the mean is 4.6. The distances of the data points to the mean are 1.6, 1.6, 0.4, 1.4, and 1.4. The MAD is $\dfrac{1.6 + 1.6 + 0.4 + 1.4 + 1.4}{5} = 1.28$
measure of center	A <u>measure of center</u> is a statistic describing the middle of a data set. The mean, the median, and the mode are three commonly used measures of center of a numerical data set.
measure of spread	A <u>measure of spread</u> is a statistic describing the variability of a data set. It describes how far the values in a data set are from the mean or median. The standard deviation, the mean absolute deviation (MAD), and the interquartile range (IQR) are three measures of spread of a numerical data set.
median	The <u>median</u> of a data set is a measure of center equal to the middle number in the data set, when the values are placed in order from least to greatest. If there is an even number of values in the data set, the median is taken to be the mean (average) of the two middle values. The median of the data set 1, 1, 1, 3, 5, 5, 6, 7, 23 is 5, since the first 5 is the middle value. The median of the data set 5, 6, 7, 23 is the mean (average) of the two middle numbers, (6 + 7) ÷ 2 = 6.5, which is the average of 6 and 7.

Word or Phrase	Definition
mode	The <u>mode</u> of a data set is the value(s) that occur(s) most often. A data set may have more than one mode. It may also have no mode if all values occur the same number of times. The mode of the data set 1, 1, 1, 3, 5, 6, 6, 7, 23 is 1, since the data value 1 occurs more frequently than any other data value. If a 6 were added to this data set, 6 would also be a mode.
outlier	An <u>outlier</u> of a data set is a data value that is a striking deviation from the overall pattern of values in the data set. For the data set 1, 1, 1, 3, 5, 6, 6, 7, 23, the data value 23 is a potential outlier. It appears unusually large relative to the other data values.
quartiles	The <u>quartiles</u> of a data set are points that divide the data set into four equally sized groups, when the values are placed in order from least to greatest. The <u>second quartile</u> is the median, denoted by Q_2. The <u>first quartile</u>, denoted by Q_1, is the median of the lower half of the data set (the data values less than the middle data value), and the <u>third quartile</u>, denoted by Q_3, is the median of the upper half of the data set. Given the ordered data set 1, 1, 1, 3, 5, 5, 6, 7, 23, • The middle value is the first 5: Median = 5. This is also the second quartile Q_2, • The lower half of the data set is 1, 1, 1, 3. Therefore $Q_1 = 1$. • The upper half of the data set is 5, 6, 7, 23. Therefore, $Q_3 = 6.5$.
range (of a data set)	The <u>range</u> of a numerical data set is the difference between the greatest and least values in the data set. The range of the data set 1, 1, 1, 3, 5, 5, 6, 7, 23 is 22, since 22 = 23 – 1.
statistical question	A <u>statistical question</u> is a question where numerical data that has potential for variability can be collected and analyzed for the purpose of answering the question. A statistical question: "How much TV do students in my class watch on average?" NOT a statistical question: "How many hours of TV did you watch last week?"

Finding Measures of Center

Here are the number of siblings for 13 different students:

$$3, \ 4, \ 5, \ 2, \ 2, \ 3, \ 3, \ 2, \ 2, \ 5, \ 7, \ 1, \ 1$$

> This next part of Student Resources includes examples and explanations for use in class or at home.

To find the <u>median</u>, order the value from least to greatest and find the middle number. number of values in the data set, the median is the mean (average) of the two middle numbers.

The median for the siblings data set: 1, 1, 2, 2, 2, 2, ③ 3, 3, 4, 5, 5, 7

To find the <u>mode</u>, find the value(s) that occur(s) most often.

The mode for the siblings data set: the value of 2 occurs most often.

To find the <u>mean</u> (average) of a data set, add all the values in the data set and divide it by the number of values (number of observations, n).

Number of observations: $n = 13$

The mean for the siblings data set: $\dfrac{3 + 4 + 5 + 2 + 2 + 3 + 3 + 2 + 2 + 5 + 7 + 1 + 1}{13} = 3.08$

Finding the Range and the Quartiles

Here are the number of siblings for 13 different students:

$$3, \ 4, \ 5, \ 2, \ 2, \ 3, \ 3, \ 2, \ 2, \ 5, \ 7, \ 1, \ 1$$

To find the <u>range</u> of a data set, find the difference between the greatest and least values in the data set.

For the siblings data set, the range is 6, since $7 - 1 = 6$

To find <u>quartiles</u>, first put the numbers in numerical order. Then locate the points that divide the set into four equal parts.

1	1	2	2	2	2	3	3	3	4	5	5	7

↑ minimum   ↑ Q_1 ↑ Q_2 median   ↑ Q_3 ↑ maximum

For the siblings data set: $Q_1 = 2$ (the 1st quartile)
$Q_2 = 3$ (the 2nd quartile)
$Q_3 = 4.5$ (the 3rd quartile)

Note that Q_1 is the median of the first half of the data set and Q_3 is the median of the second half.
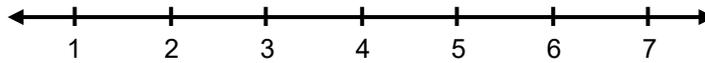
How to Construct a Dot Plot

A <u>dot plot</u> (also called a <u>line plot</u>) displays data on a number line with a dot (•) or an X to show the frequency of data values.
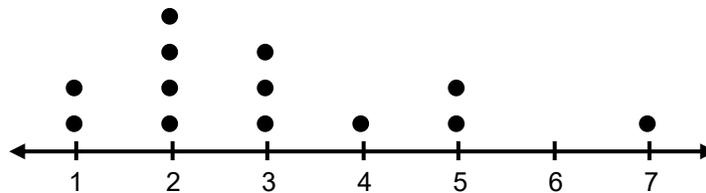
Here are the number of siblings for 13 different students:

3, 4, 5, 2, 2, 3, 3, 2, 2, 5, 7, 1, 1

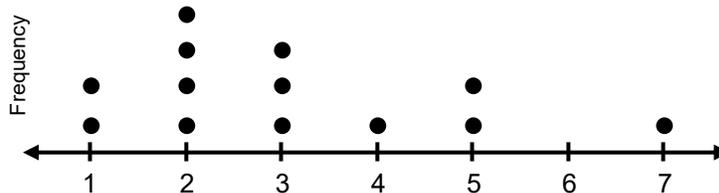1. Make a number line that extends from the minimum data value to the maximum data value.

2. Mark a dot or an X for every data value.

3. Write a title and labels.

Number of Siblings for Students in our Class

Heads Up! Be sure to line up dots or X's properly. The graph below is visually misleading in a few places. The number of dots at 4 and 5 are the same, but one set is higher than the other, possibly implying there are more. The number of dots at 2 and 4 are different, but they peak at the same height, possibly implying there are the same number of dots.
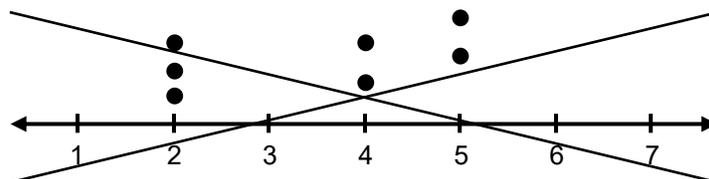
How to Construct a Histogram

A <u>histogram</u> is a data display that uses adjacent rectangles to show the frequency of data values in intervals. The height of a given rectangle shows the frequency of data values in the interval shown at the base of the rectangle.

Nancy asks each of her 21 classmates how many coins they have in their backpacks. Then she puts the data set in order.
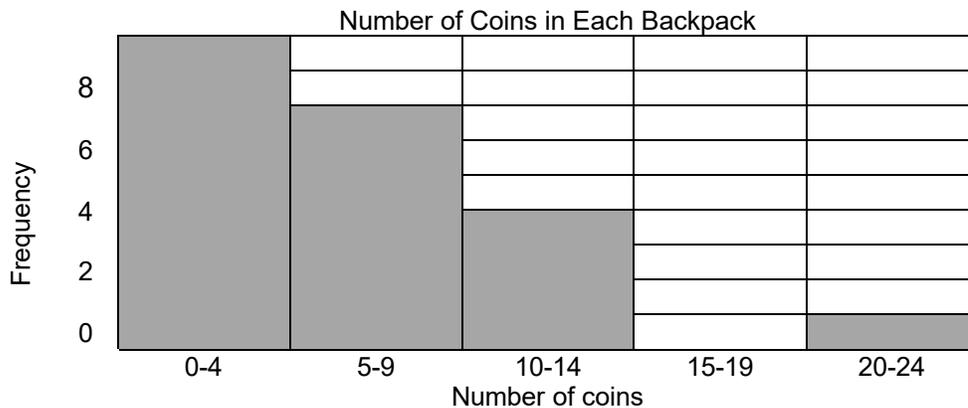
$$0, \; 0, \; 1, \; 2, \; 2, \; 2, \; 2, \; 3, \; 3, \; 5, \; 5, \; 7, \; 7, \; 7, \; 7, \; 7, \; 10, \; 10, \; 10, \; 12, \; 21$$

To construct the histogram:

1. Divide the number of coins into equally spaced intervals and make a frequency table:
    (Here we choose intervals of five.)

Intervals (number of coins)	Frequency
0-4	9
5-9	7
10-14	4
15-19	0
20-24	1

2. Record frequencies as rectangles on a data display. Add a title and label the axes.

Heads Up! Be sure to make equally spaced intervals. The graph below is visually misleading. The third column has an interval that is twice the others, but the same number of data points as the column to the left of it.
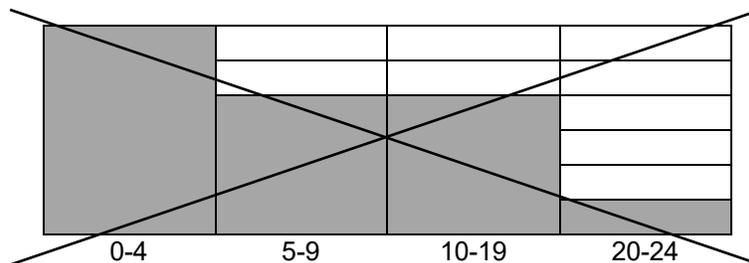
How to Construct a Box Plot

A <u>box plot</u> (or <u>box-and-whisker plot</u>) is a visual representation of the center and spread of a data set. The display is based on the five-number summary.
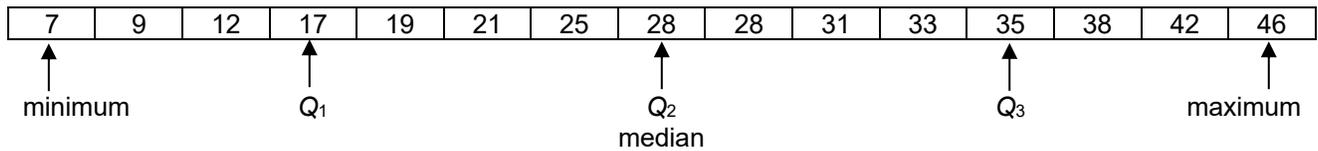
Here are the ages of 15 people:

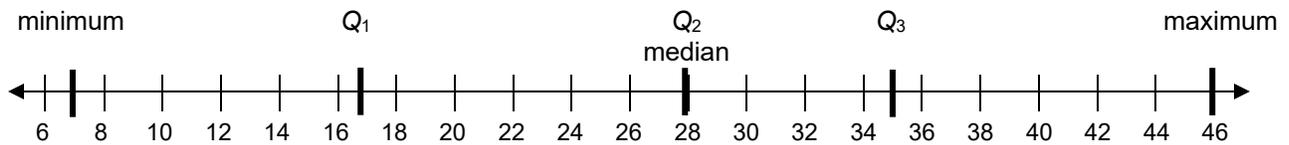21, 12, 28, 17, 46, 35, 7, 38, 42, 33, 19, 9, 31, 25, 28

1. Write the values of the data set from least to greatest.

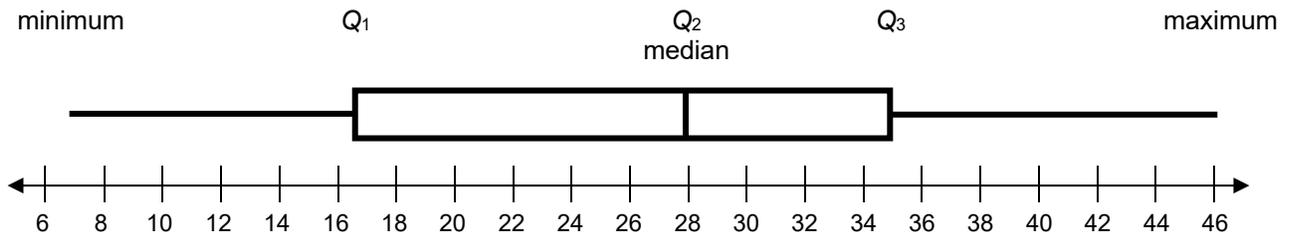7, 9, 12, 17, 19, 21, 25, 28, 28, 31, 33, 35, 38, 42, 46
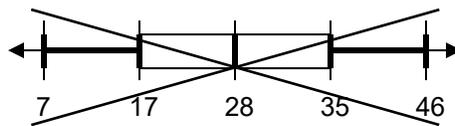
2. Find the five-number summary.

7	9	12	17	19	21	25	28	28	31	33	35	38	42	46

minimum Q_1 Q_2 Q_3 maximum
                                          median

3. Locate the five-number summary values on a number line, and indicate with vertical segments.

minimum Q_1 Q_2 Q_3 maximum
                                          median

6   8   10   12   14   16   18   20   22   24   26   28   30   32   34   36   38   40   42   44   46

4. Create a "box" to highlight the interval from the first to the third quartile, and draw "whiskers" that extend to the minimum and maximum.

minimum Q_1 Q_2 Q_3 maximum
                                          median

6   8   10   12   14   16   18   20   22   24   26   28   30   32   34   36   38   40   42   44   46

Heads Up! Be sure to scale the box and whisker plot properly. This plot is WRONG:

7   17   28   35   46

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.SP.A	**Develop understanding of statistical variability.**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
6.SP.B	**Summarize and describe distributions.**
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
6.SP.5	Summarize numerical data sets in relation to their context, such as by:
a.	reporting the number of observations.
b.	describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c.	giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d.	relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.

> The final page of a Student Packet lists all of the Common Core content and practice standards addressed in the unit. Note that some standards develop over time and are included in multiple units.

How does a mathematician plow fields? With a pro-tractor.

Parallel lines have so much in common … It's a shame they'll never meet.