Name______________________________ Period__________ Date ______________

UNIT 8
STUDENT PACKET

MathLinks
GRADE 8

LINEAR EQUATIONS AND SYSTEMS 2

Parent (or Guardian) signature ______________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

Properties of Arithmetic	
Associative property (addition and multiplication)	Commutative property (addition and multiplication)
Identity property (additive and multiplicative)	Inverse property (additive and multiplicative)
Distributive property (relating to addition and multiplication)	
Properties of Equality	
Addition property of equality (Subtraction property of equality)	Multiplication property of equality (Division property of equality)

NUMBER TRICKS

Follow your teacher's directions.
(1)

Step	Abbreviate action	Arithmetic	
a.			
b.			
c.			
d.			
e.			
f.			
g.			

- My final result is ___________
- Others in the class got ___________
- The "trick" is:

(2)

Step	Abbreviate action	Arithmetic	
a.			
b.			
c.			
d.			
e.			
f.			
g.			

- My final result is ___________
- Others in the class got ___________
- The "trick" is:

SOLVING EQUATIONS WITH RATIONAL NUMBERS

We will solve equations that involve non-integer coefficients and constants using algebra.

[8.EE.7a, 8.EE.7b; SMP3, 6, 7, 8]

GETTING STARTED

Simplify each expression.

1. $\frac{1}{5}(x - 10) + \frac{1}{10}x$	2. $\frac{1}{3}(6x + 9) - 5$	3. $-\frac{3}{4}(2x + 12) + \frac{1}{2}x$

Solve for *x*. Use any method.

4. $2(x - 1) = 2x + 1 + 3x$	5. $-2(10x + 4) = -4(6x - 8)$	6. $15x + 65 = -55 + 35x$

Solve each equation using substitution (mental math).

7. $5.6 = x + 2.3$	8. $x - 1.5 = 1.5$
9. $5.5 = 3x + 2.5$	10. $-0.2x = -0.04$
11. $x + \frac{1}{2} = 4\frac{1}{2}$	12. $x - \frac{3}{4} = 2$
13. $\frac{1}{2}x = 8$	14. $-\frac{x}{2} = 8$

CAN YOU SOLVE THESE IN TWO WAYS?

Follow your teacher's directions for (1) – (2).

(1)	
(2)	

Solve using any method.

3. $-\frac{1}{2}(x + 8) = \frac{1}{4}(x - 4)$	4. $-0.5(x - 6) = 1.5x - 1.2$

5. Do you prefer to "remove" fractions and decimals at the start of the solving process?

6. Record the meanings of the addition (subtraction) property of equality and multiplication (division) property of equality in **My Word Bank.**

PRACTICE 1

Solve each equation below using any method. Indicate if there are no solutions or infinitely many solutions.

1. $\frac{1}{2}x + 2 = \frac{1}{4}x - 6$	2. $x + \frac{3}{4} = x + \frac{5}{6}$
3. $3x - \frac{4}{5} = 2\left(x + \frac{1}{5}\right)$	4. $\frac{1}{2}(2x + 4) + x = \frac{1}{3}(6x + 6)$
5. $\frac{3}{2}(x + 4) = 2(x - 1)$	6. $\frac{1}{2}\left(\frac{1}{6}x - 1\right) = \frac{1}{4}\left(\frac{1}{3}x - 1\right)$

7. Below is some of Raj's work solving an equation. Explain his reasoning. Identify properties when appropriate. Then find the solution. Refer to **Student Resources** for the properties.

$7x - \frac{1}{8} - x = 4x + \frac{1}{4}$ $7x - x - \frac{1}{8} = 4x + \frac{1}{4}$ $6x - \frac{1}{8} = 4x + \frac{1}{4}$	Explanation:

8. Record the meanings of associative property and commutative property in **My Word Bank.**

PRACTICE 2

Solve each equation below using any method. Indicate if there are no solutions or infinitely many solutions.

1. $5x + 0.4 = 2x + 0.7$	2. $1.6x + 9.8 + 2x = 5.4 - 0.8x$
3. $-x + 2.5 = 4.6 - 2x$	4. $0.2(x + 0.3) = 0.4(x - 0.1)$
5. $-1.5x + 3 + 3x = 2(x - 2.5)$	6. $-0.4(x - 2) + 0.2 = -0.2(x - 1) - 0.5$

7. Below is some of Taj's work solving an equation. Explain her reasoning. Identify properties when appropriate. Then find the solution. Refer to **Student Resources** for the properties.

$2x + 3.6 = 3(x + 2.5)$ $2x + 3.6 = 3x + 7.5$ $\underline{-2x} \quad = \underline{-2x}$ $0 + 3.6 = x + 7.5$ $3.6 = x + 7.5$	Explanation:

8. Record the meanings of <u>distributive property</u>, <u>identity property</u>, and <u>inverse property</u> in **My Word Bank.**

SOLVING SYSTEMS USING ALGEBRA

We will use substitution to solve systems of equations. We will also explore the elimination method.

[8.EE.8a, 8.EE.8b, 8.F.3; SMP6, 7, 8]

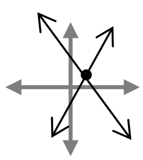
GETTING STARTED

1. Explain what it means for a number to be a solution to a linear equation in one variable.

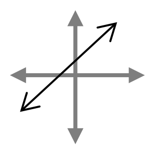
2. Explain what it means for an ordered pair of numbers to be a solution to a system of linear equations.

Recall that a system of two linear equations—when graphed—can look something like:

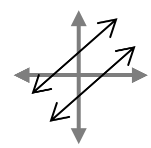
A



B



C



For each description below, write how many solutions each has, and what form solutions take (a number or an ordered pair of numbers).

3. A single linear equation in one variable, like $10 = 2x + 8$.

4. A single linear equation in two variables, like $y = 2x + 8$.

5. A system of two linear equations that intersect in one point, like diagram ____ above.

6. A system of two linear equations that are represented by parallel lines, like diagram ____ above.

7. A system of two equivalent linear equations that are represented by coinciding lines, like diagram ____ above.

USING SUBSTITUTION TO SOLVE SYSTEMS OF EQUATIONS

Below are four systems of equations. Complete the table.

System	Use substitution to write one equation in *x*, and then solve the equation.	Describe the solution(s) to the system.
1. $\begin{cases} y = 2x + 8 \\ y = -3x - 2 \end{cases}$		
2. $\begin{cases} y = 2x + 8 \\ y = 2x - 5 \end{cases}$		
3. $\begin{cases} y = 2x + 8 \\ y = 2(x + 4) \end{cases}$		
4. $\begin{cases} y = x - 5 \\ 2x + y = 4 \end{cases}$		

In Unit 7, **Practice 4**, we graphed the systems below and estimated their solutions because it was difficult to see on the graph where they intersected.

For each system below, use substitution to write an equation in one variable, solve the equation, and write the solution(s) to the system.

5. $\begin{cases} y = -4x + 2 \\ y = 8x - 1 \end{cases}$	6. $\begin{cases} y - 2 = x \\ 6x + y = -4 \end{cases}$

PRACTICE 3

Solve each system below by substitution. Check. Write the solution as an ordered pair or describe the solution(s).

1. $\begin{cases} y = 2x + 12 \\ 4x = y \end{cases}$	2. $\begin{cases} x = y + 20 \\ x = 5y \end{cases}$
3. $\begin{cases} 3x + y = 12 \\ y - 2 = -5x \end{cases}$	4. $\begin{cases} -y = -1 - 2x \\ 8x + 4 = 4y \end{cases}$
5. $\begin{cases} y - 2x = 6 \\ 2x + 2 = y \end{cases}$	6. $\begin{cases} y = 3x + 1 \\ x = -y + 7 \end{cases}$

SOLVING SYSTEMS BY ELIMINATION

Follow your teacher's directions for (1) – (5).

(1) $\begin{cases} x + y = 30 \\ x - y = 12 \end{cases}$	(2) $\begin{cases} 3x + y = 2 \\ x - 2y = 3 \end{cases}$
(3)	
(4)	(5)

6. Assemble the **Big Square Puzzle: Linear Equations and Systems 2** as directed by your teacher. Then find a system where the sum of the solutions is 0 (i.e., $x + y = 0$). Write the system and show how you found the solutions.

PRACTICE 4

Solve each system below algebraically. Check. Write the solution as an ordered pair or describe the solution(s).

1. $\begin{cases} x + y = 20 \\ -x + y = 12 \end{cases}$	2. $\begin{cases} 4x + y = 6 \\ 2x + 2y = 9 \end{cases}$
3. $\begin{cases} x + y = 12 \\ 3x - y = 20 \end{cases}$	4. $\begin{cases} 3x = y + 3 \\ 2y = 5x - 1 \end{cases}$
5. $\begin{cases} 4x - 3y = 12 \\ 3y = 4x - 6 \end{cases}$	6. $\begin{cases} 2y = 5x - 1 \\ 2 - 10x = -4y \end{cases}$

PRACTICE 5

Solve each system below algebraically. Check. Write the solution as an ordered pair or describe the solution(s).

1. $\begin{cases} y = 1.6x + 12.4 \\ 2.4x = y \end{cases}$	2. $\begin{cases} x = y + \frac{5}{2} \\ \frac{1}{2}x = y \end{cases}$
3. $\begin{cases} y = 3.5x - 1.5 \\ x = -y + 7.5 \end{cases}$	4. $\begin{cases} -0.1y = -0.1 - 0.2x \\ 0.5(8x + 8) = 2y \end{cases}$

Explain how you would solve each system below. Do not solve.

5. $\begin{cases} 2x - y = 0 \\ x + y = 2 \end{cases}$	6. $\begin{cases} y = 4x \\ y = x + 3 \end{cases}$

REASONING ABOUT SYSTEMS

Try to solve the systems below by using reasoning or minimal work. Check using an algebraic method if desired. Explain how you know you're correct.

1. $\begin{cases} y = x \\ y = x + 2 \end{cases}$	2. $\begin{cases} y = 5(x + 10) \\ y = 5x + 50 \end{cases}$
3. $\begin{cases} y = 2x - 9 \\ 3y = 3(2x - 9) \end{cases}$	4. $\begin{cases} y = -3x - 6 \\ y = -3x + 6 \end{cases}$
5. $\begin{cases} y = 2x \\ y = x + 2 \end{cases}$	6. $\begin{cases} y = 4x \\ y = x + 3 \end{cases}$

ALGEBRA APPLICATIONS

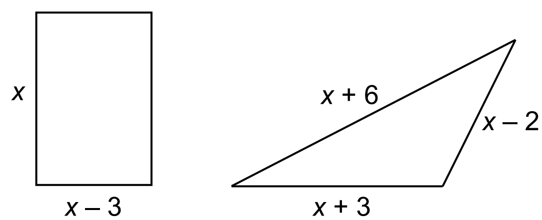
We will solve geometry, rate, and money problems using algebra.

[8.EE.7b, 8.EE.8c, 8.F2, 8.F.3, 8.F.4; SMP1, 2, 3, 4, 6]

GETTING STARTED

Solve the problems below using the organizational structure provided.

1. These figures have the same perimeter. What is the perimeter of each?



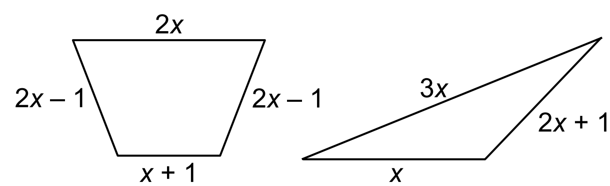
Let x =

Write one equation in x and solve:

Rectangle perimeter:

Triangle perimeter:

2. These figures have the same perimeter. What is the perimeter of each?



Let x =

Write one equation in x and solve:

Trapezoid perimeter:

Triangle perimeter:

WATERING CANS

Follow your teacher's directions.

<table>
<tr><td>(1) Facts and questions:</td><td rowspan="2">(3)</td></tr>
<tr><td>(2) Let x =
Equation:
Solution:
Answer:</td></tr>
<tr><td>(4)</td><td>(5)</td></tr>
</table>

PRACTICE 6

For each problem, define a variable, set up an equation, solve the equation, and answer the question.

1. Ada has \$84.75 and is saving \$58.50 per week. Thabo has \$177.25 and is saving \$40 per week. After how many weeks will they have the same amount of money?	2. Hadiza opens a savings account and starts to deposit 20% of her \$1,800 monthly earnings every month. At the same time, she is paying \$740 per month in bills from her checking account that has \$6,600 in it. After how many months will the two accounts have the same amount of money in them?
3. A yellow hot air balloon is 750 feet above the ground and rising at a constant rate of 3 feet per second. A blue hot air balloon starts on the ground and is rising at a constant rate of 8 feet per second. How long with it take for the blue balloon to reach the same altitude as the yellow balloon?	4. A green hot air balloon was at the maximum allowable 3,000 feet above the ground and began to descend at a constant rate of 10 feet per second. At the same time, a red hot air balloon at 300 feet above the ground starts to rise at a constant rate of 5 feet per second. How long with it take for the two balloons to be at the same altitude?

TALIA'S COIN JAR

Follow your teacher's directions.

(1)

(2)

(3) – (4)

PRACTICE 7

Talia's friends, Maya and Mateo, also have coin jars. Use a structure similar to **Talia's Coin Jar** to solve each problem below.

1. Maya's coin jar contains 46 dimes and nickels totaling \$3.25.	2. Mateo's coin jar contains 54 nickels and quarters totaling \$6.90.
3. Maya gets another jar where there are twice as many pennies as quarters. The value of those coins is \$5.40. Find the number of pennies and quarters using substitution.	4. Mateo gets another jar where there are three times as many nickels as dimes. The value of those coins is \$3.75. Find the number of pennies and quarters using substitution.

PRACTICE 8

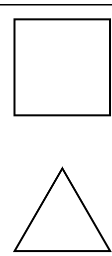
Use a structure similar to **Talia's Coin Jar** to solve each problem below.

1. There are pigs and chickens on the farm. There are 65 heads and 226 legs. How many chickens are there?

2. There are bicycles and tricycles at the park. There are 82 handlebars and 189 wheels. How many tricycles are there?

For each problem below, define a variable and set up and solve an equation. Then explain why the solution does not make sense in the context of the problem.

3. Suppose the square and the equilateral triangle to the right have the same side lengths and the same perimeters. How long is the length of each side?



   Equation and solution:

   Explanation:

4. Keiko has $400 and is saving $40 per week. Dev has $300 and is saving $30 per week (at the same time). In how many weeks will each person have the same amount saved?

   Equation and solution:

   Explanation:

TRAINING FOR A MARATHON

Marathon runners keep track of their progress by measuring “pace” (minutes per mile). Robin and Jacob are training for an upcoming marathon. They don’t usually train together because their paces are so different, but decide to train together today. Jacob says, “I’ll give you a one-hour head start. Let’s see when I catch up to you.”

Robin is an average runner. Below is a table of his training at a constant pace.

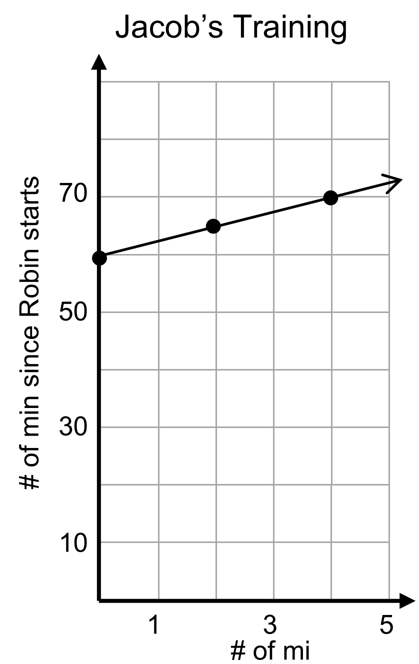

Robin

# of miles (*x*)	0	1	2	3	4	5
# of minutes (*y*)	0	10	20	30	40	50


Jacob

Jacob is an excellent wheelchair athlete.

To the right is a graph of his training at a constant pace.



1. How is Jacob’s head start reflected on the graph?

2. Write each of their initial values.

3. Write each of their paces in minutes per mile.

4. Write equations to represent each of their training paces.

5. Use substitution to solve this system. Then state the time it takes for Jacob to catch up to Robin and at what mile that occurs.

PRACTICE 9

Refer to **Training for a Marathon** to complete this page.

Another friend, Kim, wants to train for the marathon too.
He is a walker, whose pace is 15 minutes per mile.

Kim

1. List the athletes from fastest to slowest. __________, __________, __________

2. How long does it take each to go 6 miles?

   Robin _________ minutes Jacob _________ minutes Kim _________ minutes

3. How long does it take each to go *x* miles?

   Robin _________ minutes Jacob _________ minutes Kim _________ minutes

Solve the problems below based on different training sessions. Use algebra to determine when one athlete catches up to the other.

Let x = the number of miles.

4. Jacob gives Kim a 30-minute head start. Explain the solution in the context of the problem and how much time each athlete has trained.	5. Robin gives Kim a 40-minute head start. Explain the solution in the context of the problem and how much time each athlete has trained.

6. What did you find interesting or challenging about the Marathon problems?

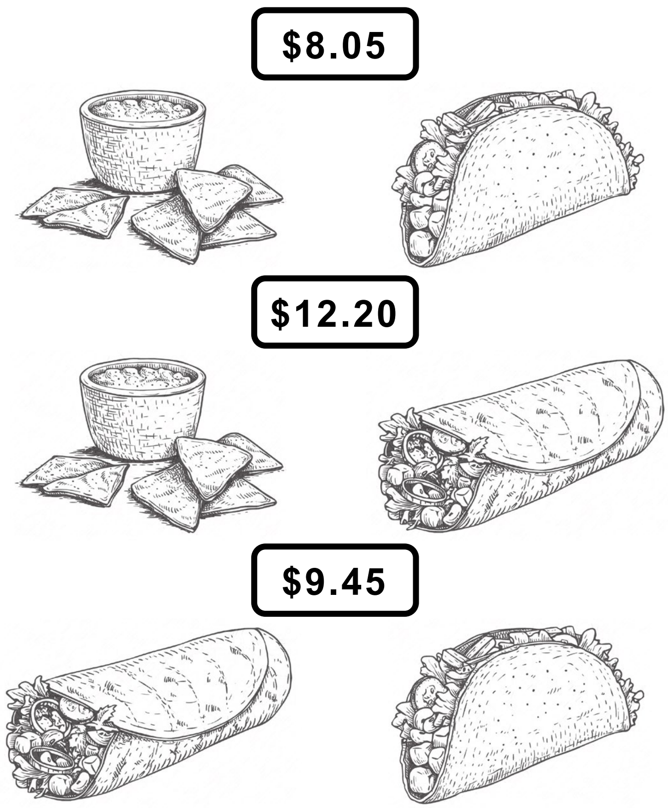
EXTEND YOUR THINKING: TACOS, BURRITOS, AND GUAC… OH MY!

The prices of the items below are given in pairs. Use any method to find the cost of each item individually. Hint: defining variables and using equations works nicely.

The cost of guacamole and a taco is $8.05.

The cost of guacamole and a burrito is $12.20.

The cost of a burrito and a taco is $9.45.



REVIEW

OPEN MIDDLE PROBLEMS: LINEAR EQUATIONS AND SYSTEMS 2

For each problem below, use the digits 1 through 9 no more than one time each.

Structure: $\square x + \square = \square x + \square$

1. Write an equation and find its solution(s).	2. Write an equation with exactly one solution between 1 and 2.
3. Write an equation with exactly one solution between -1 and -2.	4. Write an equation with exactly one solution between 1 and -1.

PLAY IT POSITIVELY AND NEGATIVELY!

For 2 – 4 players

Materials: Play it Positively and Negatively! playing cards, 1 number cube.

Rules:

- Determine prior to each roll if the number on the number cube will represent a positive or negative value. Roll the number cube. This will represent *x* (or its opposite) for the round.
- With cards in a pile, face down, each player chooses a card, substitutes the value of *x* into the equation and solves it.
- Players check each others' solutions. The player with the greatest value of *y* for the solved equation gets all the cards.
- Play continues until all cards are gone. The player with the most cards is the winner.

Record your work for each round here:

Round 1: For this round, *x* = ____. I got card # _____.	Round 2: For this round, *x* = ____. I got card # _____.
Round 3: For this round, *x* = ____. I got card # _____.	Round 4: For this round, *x* = ____. I got card # _____.

POSTER PROBLEMS: LINEAR EQUATIONS AND SYSTEMS 2

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
$\begin{cases} y - \frac{1}{3}x = 0 \\ -y = \frac{2}{3}x + 1 \end{cases}$	$\begin{cases} y + x = \frac{1}{2} \\ 2y = x \end{cases}$	$\begin{cases} 2(0.2x - 6) = y - 12 \\ 0.5(2y - 16) = 1.2x \end{cases}$	$\begin{cases} 0.2y = 0.2x - 0.4 \\ 0.6y = 0.4x - 0.6 \end{cases}$

A. Copy the problem. Write both equations in slope-intercept form.

B. Solve the system by substitution.

C. Substitute the *x*-value from part B into the equation from part B as a check.

D. Explain two things in writing:
   1. Why does substituting this this one value into the equation make the ordered pair a true solution?
   2. Why is the substitution method preferable to the graphing method for this problem?

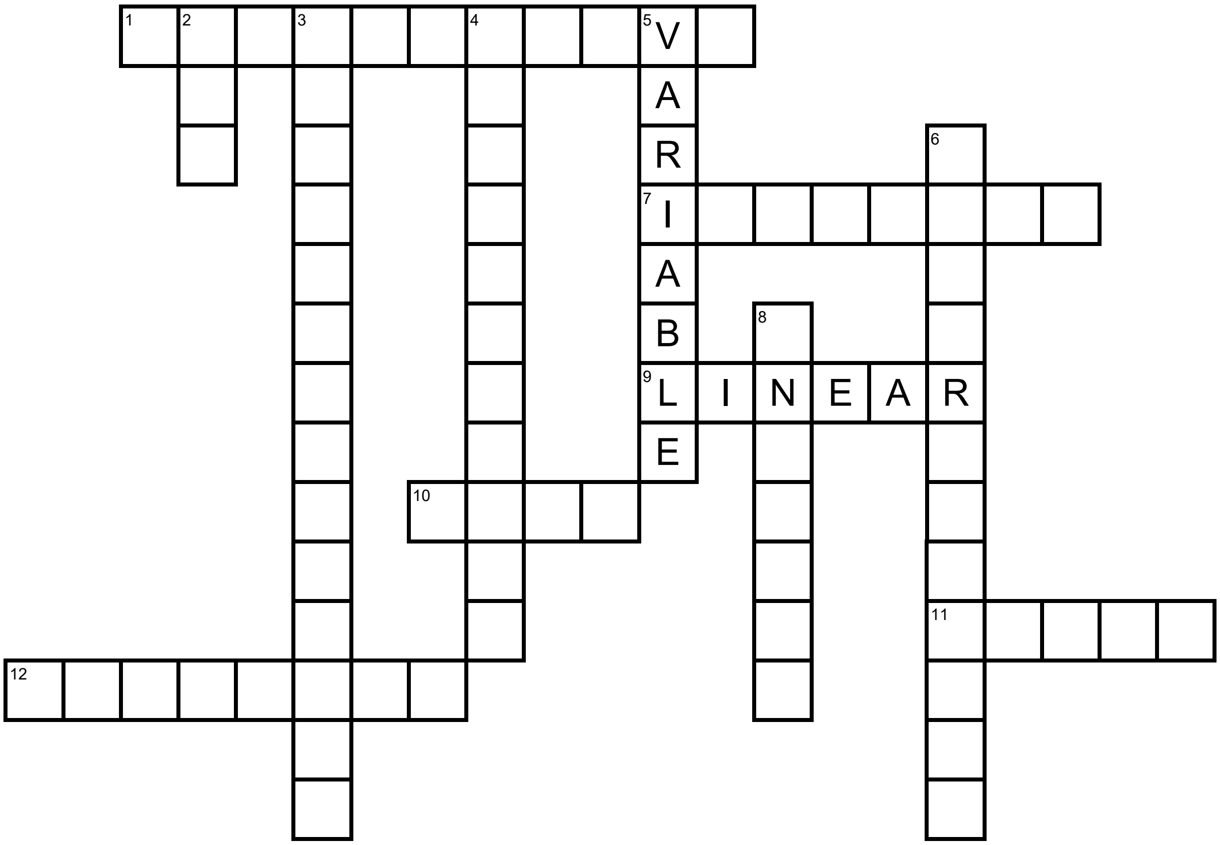
Part 3: Return to your seats. Work with your group.

For the system of equations below, write both equations in slope-intercept form and explain how you know what the solution(s) is/are without graphing.

$$\begin{cases} \frac{3}{4}\left(x - \frac{5}{6}\right) = \frac{1}{2}y \\ -0.625 = 0.5y - 0.75x \end{cases}$$

VOCABULARY REVIEW

Use the **Student Resources** as needed to review the properties of arithmetic and equality.



Statements and properties about the steps for solving this equation are shown. Use the clues to complete the crossword puzzle. The first one is done for you.

$3(3x - 2) = -4(4 - x) + 18x + 30 - 12x$	This is a (9 across) equation in one (5 down).
$9x - 6 = -16 + 4x + 18x + 30 - 12x$	(6 down) property
$9x - 6 = (-16 + 30) + (4x + 18x - 12x)$	(4 down) and (1 across) properties
$9x - 6 = 14 + 10x$	combine (10 across) (11 across)
$\underline{+6 \quad +6}$	(12 across) property of equality
$9x + 0 = 20 + 10x$	additive (8 down) property ($-6 + 6 = 0$)
$9x = 20 + 10x$	additive (7 across) property ($9x + 0 = 9x$)
$\underline{-10x \qquad -10x}$	(12 across) property of equality
$-1x = 20$	additive (8 down) and (7 across) properties
$-1 \bullet (-1x) = -1 \bullet (20)$	(3 down) property of equality
$1x = -20$	just plain ol' arithmetic
$x = -20$	Multiplication property of (2 down)

SPIRAL REVIEW

1. **Alge-Grid: What's the *a*?** Each clue gives the value of a corresponding cell. Use clues to find *a*, which has the same value in all cells. Once evaluated, the cells will contain the whole numbers 1 – 9, exactly once each.

The Alge-Grid

$a^2 - 8a - 2$	$\sqrt{a} + (a \div 3)$	$a(a \div 9)$
$a^0 + (a \div 3) - 3$	$a - 5$	$(a-8)^2 + 4$
$(8a \div 9)$	$(a + 1) \div 2 - 3$	$a \div 3$

The Clues

Number of red stripes on the U.S. flag		
	Least composite number	
	Only even prime number	

2. Solve each equation below for *y* in terms of *x*.

a. $6x + 2y = 24$	b. $2y - 2x = 30$

3. Write each number below in scientific notation. Circle the smallest value.

a. 0.00083	b. $5,520,000 \times 20,000$	c. 0.01082×0.005

4. Write each number below in standard notation. Circle the largest value.

a. 1.79×10^4	b. 31.2×10^3	c. 2.2×10^{-2}

SPIRAL REVIEW
Continued

5. Use the diagram to the right where two parallel lines are cut by a transversal.

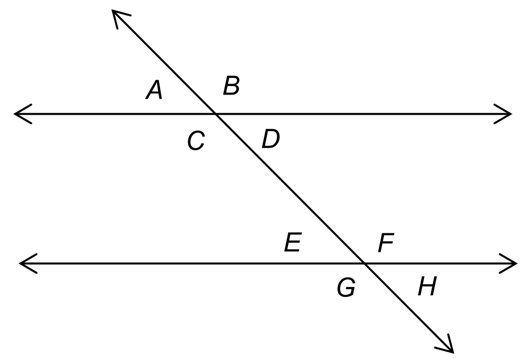
   a. List all pairs of corresponding angles.

   b. List all pairs of alternate interior angles.

   c. List all pairs of alternate exterior angles.

   d. List all pairs of vertical angles.

   e. List 4 pairs of supplementary angles.



   f. If $|\angle B| = (3y - 24)°$, $|\angle C| = x°$, and $|\angle E| = y°$ find x and y.

6. Selena has a bowl full of fruit. Find the volume of each piece of fruit. Use π = 3.14 and round to the nearest hundredth if necessary.

a. A grapefruit with a diameter of 7 cm.	b. An apricot with a radius of 1.5 cm.	c. An orange with a diameter of 5 cm.

7. A basketball court is a rectangle that measures 28 meters by 15 meters. How many meters is it diagonally across the court? Round to nearest hundredth if necessary.

SPIRAL REVIEW

Continued

8. Maura asked people at the fitness loop how many minutes they walked and how many calories they burned each day.

a. Explain a possible association between the variables.

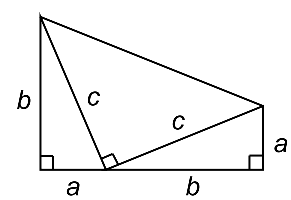
b. Draw an estimated line of best fit and write the equation of the line.

c. What does the *y*-intercept represent?

d. What does the slope represent?

e. If a person walked for one and a half hours, how many calories would they burn?

9. President James Garfield was at one time a mathematics teacher. He discovered a proof of the Pythagorean theorem in 1876. He began with a trapezoid (recall A = $\frac{1}{2}(b_1 + b_2) \bullet h$). To complete his proof:



One base is ____, the other base is ____, and height is ________.

An expression for the area of the trapezoid is ___________________________

The trapezoid is made up of three triangles. An expression for the sum of the three triangle areas is ___________________________.

Fact: $(a + b)(a + b) = a^2 + 2ab + b^2$. Equate the two area expressions and part c above and simplify.

Why does this prove the Pythagorean theorem?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity
- Extend applications of volume to cylinders, cones, and spheres
- Explore bivariate data
- Complete the real number system
- Solve linear equations in one variable and linear systems in two variables
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean theorem
- Create, analyze, and use linear functions in problem solving
- Explore exponents and roots, and very large and very small quantities

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practice.** Explain where it was important to calculate accurately [SMP6]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** What problem or page in this unit seemed most complex to you? Describe changing quantities involved, if any. Did using algebra help you tackle it?

STUDENT RESOURCES

Word or Phrase	Definition
slope-intercept form	The slope-intercept form of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. If $y = x + 5$, and we know that $x = 3$, then we may use substitution to rewrite the first equation to get $y = 3 + 5$. If $y = x + 10$, and we know also that $y = 2x + 4$, then we may use substitution to write one equation in x to get $x + 10 = 2x + 4$.
system of linear equations	A system of linear equations is a set of two or more linear equations in the same variables. An example of a system of linear equations in x and y: $\begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1 \quad (a \neq 0)$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).
For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality: If $a = b$, then $b = a$
- ✓ Transitive property of equality: If $a = b$, and $b = c$, then $a = c$

A Strategy for Solving Equations with Rational Coefficients

Equations with rational number coefficients may be solved the same way that equations with integer coefficients are solved using properties of arithmetic and equality. However, many people prefer to rewrite the equation without fractions or decimals before solving it. This can be accomplished by:

- Determining a common multiple for all denominators in the equation.
- Multiplying both sides of the equation by that common multiple.

What remains will be an equation to solve with integer coefficients.

$\frac{2}{3}(x-1) = \frac{1}{6}(x+2)$ $6\left[\frac{2}{3}(x-1)\right] = 6\left[\frac{1}{6}(x+2)\right]$ $4(x-1) = x + 2$	A common multiple of 3 and 6 is 6. Here we multiply both sides of the equation by 6 (multiplication property of equality). The result is an equation with integer coefficients.
$0.1x + 0.25(33 - x) = 5.1$ $100\ [0.1x + 0.25(33 - x)] = 100\ [5.1]$ $10x + 25(33 - x) = 510$	This equation includes tenths ($\frac{1}{10}$) and hundredths ($\frac{1}{100}$). A common multiple of these denominators is 100. Here we multiply both sides of the equation by 100. The result is an equation with integer coefficients.

Solving a System of Linear Equations by Substitution

While there may be multiple ways to use substitution to solve a system of equations, this is one way that has been demonstrated in this unit.

1. Write both equations in slope-intercept form.

$$\begin{cases} 2x + y = 5 \quad \rightarrow \quad y = -2x + 5 \\ x + 2y = 4 \quad \rightarrow \quad 2y = -x + 4 \quad \rightarrow \quad y = -\frac{1}{2}x + 2 \end{cases}$$

2. Use the substitution property. Since the right side of both equations are equal to the same thing, namely y, the two expressions in x must be equal to each other.

   Write one equation and solve for x.

$$-2x + 5 = -\frac{1}{2}x + 2$$
$$x = 2$$

3. Substitution this x-value into either equation to obtain the y-value.

$$y = -2(2) + 5$$
$$y = 1$$

Solution to the system: (2, 1)

For this example, another substitution approach would be to write the first one equation in slope-intercept form and substitute the expression for y in the second equation. Using this approach:

$$x + 2(-2x + 5) = 4$$
$$x = 2$$

Solving a System of Linear Equations by Elimination

Elimination, which applies the multiplication property of equality and the addition property of equality, is another method for solving a system of equations. Here is an example.

Example: Solve this system of equations by elimination.
(We will number each equation with brackets to keep track of them.)

$$\begin{cases} 2x + y = 5 & [1] \\ x + 2y = 4 & [2] \end{cases}$$

1. Use the multiplication property of equality. Multiply both sides of one (or both) equations by some number that will make one of the variable expressions in each equation opposites of each other. In this case, we might multiply both sides of the first equation by -2.

$$-2(2x + y) = -2(5) \rightarrow -4x - 2y = -10 \quad [3]$$

2. Use the addition property of equality. Add expressions on each side of the equation together. Solve.

$$\begin{array}{rcll} -4x - 2y &=& -10 & [3] \\ \underline{x + 2y} &=& \underline{4} & \underline{[2]} \\ -3x &=& -6 & \\ x &=& 2 & \end{array}$$

3. Substitute into one of the original equations to find y.

$$[1] \quad 2x + y = 5 \rightarrow 2(2) + y = 5$$

$$y = 1$$

4. Substitute into the other original equation to check.

$$[2] \quad x + 2y = 4 \quad 2 + 2(1) = 4 \quad \text{(true)}$$

A Strategy for Organizing Problem Solving Work Involving Equations

Many algebra problems can be solved with these steps. Here are the steps and an example.

> Talia's coin jar contains nickels and quarters. There are 38 coins in all. The total value of jar is $5.30. Find the coins in the jar.

• Identify the variable(s)	Let n = the number of nickels Let q = the number of quarters
• Write the equation(s)	$n + q = 38 \rightarrow q = 38 - n$ $0.05n + 0.25q = 5.30$
• Solve the equation(s)	By substitution: $0.05n + 0.25(38 - n) = 5.30$ $n = 21$ and $q = 17$
• Answer the question(s)	There are 21 nickels and 17 quarters
• Interpret the solution in the problem	Money makes sense: 21($0.05) + 17($0.25) = $5.30

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.C	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
8.EE.7	Solve linear equations in one variable:
a.	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
b.	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.EE.8	Analyze and solve pairs of simultaneous linear equations:
a.	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b.	Solve systems of two linear equations in two variables algebraically, ~~and estimate solutions by graphing the equations~~. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
c.	Solve real-world and mathematical problems leading to two linear equations in two variables.
8.F.A	**Define, evaluate, and compare functions.**
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; ~~give examples of functions that are not linear.~~ ~~*For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*~~
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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